

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

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*Isolated Singularities in Partial  
Differential Inequalities*

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**CAMBRIDGE**  
UNIVERSITY PRESS

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