

# CONTENTS

<b>0. Introduction</b> .....	9
0.1. Artin-Schreier theory and $(\varphi, \Gamma)$ -modules .....	10
0.2. Arithmetic vs. geometric .....	10
0.3. Analytic spaces associated to Banach algebras .....	11
0.4. Perfectoid fields and algebras .....	12
0.5. Robba rings and slope theory .....	13
0.6. $\varphi$ -modules and local systems .....	14
0.7. Contact with the work of Scholze .....	14
0.8. Further goals .....	16
Acknowledgments .....	17
<b>1. Algebro-geometric preliminaries</b> .....	19
1.1. Finite, flat, and projective modules .....	19
1.2. Comparing étale algebras .....	20
1.3. Descent formalism .....	23
1.4. Étale local systems .....	28
1.5. Semilinear actions .....	32
<b>2. Spectra of nonarchimedean Banach rings</b> .....	35
2.1. Seminorms on groups and rings .....	35
2.2. Banach rings and modules .....	37
2.3. The Gel'fand spectrum of a Banach ring .....	44
2.4. The adic spectrum of an adic Banach ring .....	48
2.5. Coherent sheaves on affinoid spaces .....	58
2.6. Affinoid systems .....	63
2.7. Glueing of finite projective modules .....	66
2.8. Uniform Banach rings .....	69

<b>3. Perfect rings and strict <math>p</math>-rings</b> .....	77
3.1. Perfect $\mathbb{F}_p$ -algebras .....	77
3.2. Strict $p$ -rings .....	83
3.3. Norms on strict $p$ -rings .....	88
3.4. Inverse perfection .....	92
3.5. The perfectoid correspondence for analytic fields .....	94
3.6. The perfectoid correspondence for adic Banach algebras .....	98
3.7. Preperfectoid and relatively perfectoid algebras .....	114
<b>4. Robba rings and <math>\varphi</math>-modules</b> .....	117
4.1. Slope theory over the Robba ring .....	117
4.2. Slope theory and Witt vectors .....	120
4.3. Comparison of slope theories .....	126
<b>5. Relative Robba rings</b> .....	129
5.1. Relative extended Robba rings .....	130
5.2. Reality checks .....	133
5.3. Sheaf properties .....	137
5.4. Some geometric observations .....	142
5.5. Compatibility with finite étale extensions .....	145
<b>6. <math>\varphi</math>-modules</b> .....	153
6.1. $\varphi$ -modules and $\varphi$ -bundles .....	153
6.2. Construction of $\varphi$ -invariants .....	155
6.3. Vector bundles à la Fargues-Fontaine .....	157
<b>7. Slopes in families</b> .....	167
7.1. An approximation argument .....	167
7.2. Rank, degree, and slope .....	168
7.3. Pure models .....	169
7.4. Slope filtrations in geometric families .....	174
<b>8. Perfectoid spaces</b> .....	181
8.1. Some topological properties .....	181
8.2. Adic spaces .....	183
8.3. Perfectoid spaces .....	190
8.4. Étale local systems on adic spaces .....	191
8.5. $\varphi$ -modules and local systems .....	194
8.6. A bit of cohomology .....	200
8.7. The relative Fargues-Fontaine curve .....	201
8.8. Ampleness on relative curves .....	205
8.9. $B$ -pairs .....	212
<b>9. Relative <math>(\varphi, \Gamma)</math>-modules</b> .....	215
9.1. The pro-étale topology for adic spaces .....	215

9.2. Perfectoid subdomains .....	218
9.3. $\varphi$ -modules and local systems .....	222
9.4. Comparison of cohomology .....	225
9.5. Comparison with classical $p$ -adic Hodge theory .....	227
<b>Bibliography</b> .....	<b>231</b>