

MEMOIRS

ISSN 0065-9266 (print)
ISSN 1947-6221 (online)

of the
American Mathematical Society

Number 1634

The Measures with L^2 -Bounded Riesz Transform and the Painlevé Problem

Damian Dąbrowski
Xavier Tolsa

May 2026 • Volume 321 • Number 1634 (first of 6 numbers)

Contents

Chapter 1. Introduction	1
Chapter 2. Preliminaries	9
2.1. The modified dyadic lattice of David and Mattila	9
2.2. $\mathcal{P}$ -doubling cubes, cubes dominated from below, and the Main Propositions	18
Chapter 3. Tractable Trees	27
3.1. The cubes with moderate decrement of Wolff energy and the associated tractable trees	27
3.2. The Riesz transform on the tractable trees: The approximating measures η , ν , and the variational argument	35
3.3. Lower estimates for the Haar coefficients of $\mathcal{R}\mu$ for cubes near a typical tractable tree	56
Chapter 4. The proof of the first main proposition	83
4.1. The corona decomposition and the Main Lemma	83
4.2. The layers F_j^h and L_j^h	85
4.3. The proof of Main Lemma 4.1	93
4.4. The proof of the First Main Proposition 2.15	101
Chapter 5. The proof of the second main proposition	123
5.1. The good dominating family GDF	123
5.2. The layers F_j^h and L_j^h and the typical tractable trees	129
5.3. The proof of the Second Main Proposition 2.16	133
Appendix A. Parameters and families	135
A list of parameters	135
A list of cube families and related objects	136
Bibliography	139