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The Complexity of Conjugacy, Orbit Equivalence, and von Neumann Equivalence of Actions of Nonamenable Groups

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