
Contents

Preface	xi
Chapter 1. Hilbert space basics	1
§1.1. Inner product spaces and Hilbert spaces	1
§1.2. Linear functionals and the Riesz representation theorem	7
§1.3. Bounded linear transformations	8
§1.4. The trace and the Hilbert-Schmidt inner product	14
§1.5. The spectral theorem	20
§1.6. The functional calculus for normal operators	25
§1.7. Quantum mechanics: measurement and dynamics	29
Exercises	32
Chapter 2. Tensor products of Hilbert spaces	35
§2.1. Tensor products	35
§2.2. Partial traces	42
§2.3. Measurements on multicomponent systems and entanglement	45
§2.4. Purification	51
§2.5. The singular value decomposition	55
§2.6. The block matrix picture for $\mathcal{B}(\mathcal{H} \otimes \mathcal{K})$	60
§2.7. Tensor powers of operators	64
§2.8. Symmetric and antisymmetric tensor products	66
Exercises	72

Chapter 3. Monotonicity and convexity for operators	75
§3.1. Monotonicity and convexity on the real line	75
§3.2. Operator monotonicity	79
§3.3. Operator convexity	88
§3.4. Pinching and the operator Jensen inequality	89
§3.5. The Golden-Thompson inequality	92
Exercises	94
Chapter 4. von Neumann algebras on finite-dimensional Hilbert spaces	97
§4.1. Operator algebras	97
§4.2. Projections in von Neumann algebras	100
§4.3. Commutants	103
§4.4. The structure theorem	105
§4.5. Weyl-Heisenberg groups on finite-dimensional Hilbert spaces	110
§4.6. Conditional expectations in von Neumann algebras	113
§4.7. Quantum state measurement and Naimark's theorem	116
§4.8. Representations of von Neumann algebras	124
Exercises	128
Chapter 5. Positive linear maps and quantum operations	131
§5.1. Positive linear maps	131
§5.2. k -positivity and complete positivity	133
§5.3. Quantum operations	137
§5.4. Choi's theorem and Kraus representations	138
§5.5. Stinespring's factorization of CP maps	145
§5.6. Inequalities for 2-positive maps	150
§5.7. The mean ergodic theorem	156
§5.8. The set of invariant states for unital CP maps	160
§5.9. The no-cloning theorem	165
§5.10. Inequalities for positive maps	170
Exercises	174
Chapter 6. Some basic trace function inequalities	181
§6.1. The above the tangent plane inequality and subgradients	181
§6.2. The Fenchel-Moreau theorem	186
§6.3. The Schatten p -norms	189

§6.4. Noncommutative L^p norms associated to a density matrix	199
§6.5. The Powers-Størmer inequality	205
§6.6. Fidelity and Uhlmann's theorem	210
§6.7. Klein's inequality and the Peierls-Bogoliubov inequality	215
§6.8. Hölder's inequality for Schatten norms in general form	216
Exercises	222
Chapter 7. Fundamental entropy inequalities	227
§7.1. Classical entropy	227
§7.2. Quantum entropy	243
§7.3. Gibbs variational principle	251
§7.4. Measured relative entropies	255
§7.5. Strong subadditivity of the von Neumann entropy	261
§7.6. The data processing inequality	268
Exercises	269
Chapter 8. Consequences and refinements of SSA	273
§8.1. The Alicki-Fannes inequality	273
§8.2. SSA by a method of Petz	275
§8.3. SSA and the quantum Markov property	281
§8.4. Cases of equality in SSA	283
§8.5. SSA and the Peierls-Bogoliubov and Golden-Thompson inequalities	285
Exercises	289
Chapter 9. Quantification of entanglement	291
§9.1. Quantum teleportation and entanglement as a resource	291
§9.2. Entanglement of mixed states	295
§9.3. Relative entropy of entanglement	299
§9.4. Entanglement of formation	305
§9.5. Symmetric extendibility	311
§9.6. Squashed entanglement	315
§9.7. Extended SSA	325
Exercises	326
Chapter 10. Convexity, concavity, and monotonicity	327
§10.1. A tracial characterization of Schwarz maps	327
§10.2. Inequalities for Schwarz maps	331

§10.3. The Wigner-Yanase theorem and the conjectures it inspired	336
§10.4. The Lieb concavity theorem and its consequences	338
§10.5. The Epstein concavity theorem and its consequences	339
§10.6. The Lieb convexity theorem and its consequences	344
§10.7. Convexity and concavity of $A \mapsto \text{Tr}[(K^* A^p K)^s]$	348
§10.8. Quantum Rényi relative entropy inequalities	351
Exercises	360
Chapter 11. Majorization methods	365
§11.1. Majorization in $\mathbb{R}^n$	365
§11.2. Majorization and double stochasticity	366
§11.3. Majorization and convexity	370
§11.4. Ky Fan's formula and matrix majorization	371
§11.5. The Courant-Fischer theorem and matrix majorization	373
§11.6. Log-majorization	376
§11.7. Weyl's method	378
§11.8. Multivariate ALT inequalities	383
Exercises	386
Chapter 12. Tomita-Takesaki theory and operator inequalities	389
§12.1. The GNS construction and purification	389
§12.2. The Tomita-Takesaki theory	394
§12.3. The relative modular operator	399
§12.4. Trace inequalities without traces	401
Exercises	404
Appendix A. Convex geometry	407
§A.1. Convex sets	407
§A.2. Extreme points	411
§A.3. Convex functions and their Legendre transforms	416
Appendix B. Complex interpolation	425
§B.1. The three lines theorem and Hirschman's lemma	425
§B.2. Hirschman's refined three lines theorem	426
§B.3. Applications of Hirschman's lemma	430
§B.4. An alternate form for the derivative of the logarithm	432
Bibliography	435
Index	447