

Contents

Chapter 1. Introduction	1
1.1. Motivation and state-of-the-art	1
1.2. Deformation space of MGHC AdS manifolds	2
1.3. Para-hyper-Kähler structures	2
1.4. Parameterizations of $\mathcal{MGH}(\Sigma)$	3
1.5. The circle action	5
1.6. The character variety	6
1.7. Outline of techniques and proofs	8
Chapter 2. Preliminaries on Anti-de Sitter geometry	13
2.1. Maximal surfaces	13
2.2. Cotangent bundle of Teichmüller space	14
2.3. Mess' parameterization	16
2.4. Constant curvature surfaces and circle action	17
2.5. An equivalent model for $\mathcal{MS}(\Sigma)$	18
Chapter 3. The toy model: Genus 1	21
3.1. Space of linear almost-complex structures	21
3.2. The tangent space of $T^*\mathcal{J}(\mathbb{R}^2)$	22
3.3. A para-hyperKähler structure on $T^*\mathcal{J}(\mathbb{R}^2)$	23
3.4. Liouville form on $T^*\mathcal{J}(\mathbb{R}^2)$	25
3.5. Relation with $\mathcal{MGH}(T^2)$	27
3.6. A formal Mess homeomorphism	29
3.7. The circle action	33
3.8. A one-parameter family of maps	35
Chapter 4. The general case: Genus ≥ 2	37
4.1. The group of (Hamiltonian) symplectomorphisms and its Lie algebra	37
4.2. The Teichmüller space as a symplectic quotient	38
4.3. The construction of $\mathcal{MS}_0(\Sigma, \rho)$	42
4.4. The para-hyperKähler structure of $\mathcal{MS}_0(\Sigma, \rho)$	44
4.5. The proof of Theorem L	46
4.6. The proof of Proposition K	56
Chapter 5. Geometric interpretations	65
5.1. The cotangent bundle parametrization	65
5.2. The Mess homeomorphism	66
5.3. The circle action on $\mathcal{MS}(\Sigma)$	67
5.4. Para-complex geometry of the $\mathbb{P}\mathrm{SL}(2, \mathbb{B})$ -character variety	69

Chapter 6. Symplectic reduction	79
6.1. The moment maps on $T^*\mathcal{J}(\mathbb{R}^2)$	79
6.2. Donaldson's construction	81
6.3. The moment maps on $T^*\mathcal{J}(\Sigma)$	83
6.4. The symplectic quotient	86
Appendix A. Para-complex geometry	101
Bibliography	105