

FOURIER ANALYSIS

T.W. KÖRNER

Trinity Hall, Cambridge



CAMBRIDGE
UNIVERSITY PRESS

CONTENTS

PREFACE	<i>page xi</i>
PART I FOURIER SERIES	1
1 Introduction	3
2 Proof of Fejér's theorem	6
3 Weyl's equidistribution theorem	11
4 The Weierstrass polynomial approximation theorem	15
5 A second proof of Weierstrass's theorem	19
6 Hausdorff's moment problem	21
7 The importance of linearity	24
8 Compass and tides	28
9 The simplest convergence theorem	32
10 The rate of convergence	35
11 A nowhere differentiable function	38
12 Reactions	42
13 Monte Carlo methods	46
14 Mathematical Brownian motion	50
15 Pointwise convergence	56
16 Behaviour at points of discontinuity I	59
17 Behaviour at points of discontinuity II	62
18 A Fourier series divergent at a point	67
19 Pointwise convergence, the answer	74
PART II SOME DIFFERENTIAL EQUATIONS	77
20 The undisturbed damped oscillator does not explode	79
21 The disturbed damped linear oscillator does not explode	83
22 Transients	88
23 The linear damped oscillator with periodic input	93
24 A non-linear oscillator I	99
25 A non-linear oscillator II	104

26	A non-linear oscillator III	113
27	Poisson summation	116
28	Dirichlet's problem for the disc	121
29	Potential theory with smoothness assumptions	124
30	An example of Hadamard	131
31	Potential theory without smoothness assumptions	134
PART III ORTHOGONAL SERIES		143
32	Mean square approximation I	145
33	Mean square approximation II	150
34	Mean square convergence	155
35	The isoperimetric problem I	159
36	The isoperimetric problem II	166
37	The Sturm–Liouville equation I	170
38	Liouville	175
39	The Sturm–Liouville equation II	179
40	Orthogonal polynomials	185
41	Gaussian quadrature	191
42	Linkages	197
43	Tchebychev and uniform approximation I	201
44	The existence of the best approximation	207
45	Tchebychev and uniform approximation II	212
PART IV FOURIER TRANSFORMS		219
46	Introduction	221
47	Change in the order of integration I	226
48	Change in the order of integration II	230
49	Fejér's theorem for Fourier transforms	240
50	Sums of independent random variables	245
51	Convolution	253
52	Convolution on \mathbb{T}	259
53	Differentiation under the integral	265
54	Lord Kelvin	270
55	The heat equation	274
56	The age of the earth I	282
57	The age of the earth II	285
58	The age of the earth III	289
59	Weierstrass's proof of Weierstrass's theorem	292
60	The inversion formula	295
61	Simple discontinuities	300
62	Heat flow in a semi-infinite rod	308

63	A second approach	315
64	The wave equation	324
65	The transatlantic cable I	332
66	The transatlantic cable II	335
67	Uniqueness for the heat equation I	338
68	Uniqueness for the heat equation II	344
69	The law of errors	347
70	The central limit theorem I	349
71	The central limit theorem II	357

PART V FURTHER DEVELOPMENTS 363

72	Stability and control	365
73	Instability	368
74	The Laplace transform	372
75	Deeper properties	379
76	Poles and stability	386
77	A simple time delay equation	395
78	An exception to a rule	403
79	Many dimensions	407
80	Sums of random vectors	413
81	A chi squared test	418
82	Haldane on fraud	425
83	An example of outstanding statistical treatment I	429
84	An example of outstanding statistical treatment II	434
85	An example of outstanding statistical treatment III	436
86	Will a random walk return?	443
87	Will a Brownian motion return?	451
88	Analytic maps of Brownian motion	455
89	Will a Brownian motion tangle?	461
90	La Famille Picard va á Monte Carlo	467

PART VI OTHER DIRECTIONS 471

91	The future of mathematics viewed from 1800	473
92	Who was Fourier? I	475
93	Who was Fourier? II	478
94	Why do we compute?	481
95	The diameter of stars	484
96	What do we compute?	488
97	Fourier analysis on the roots of unity	491
98	How do we compute?	497
99	How fast can we multiply?	500

100	What makes a good code?	503
101	A little group theory	506
102	A good code?	509
103	A little more group theory	513
104	Fourier analysis on finite Abelian groups	519
105	A formula of Euler	525
106	An idea of Dirichlet	532
107	Primes in some arithmetical progressions	539
108	Extension from real to complex variable	546
109	Primes in general arithmetical progressions	552
110	A word from our founder	558
Appendix A: The circle \mathbb{T}		560
Appendix B: Continuous function on closed bounded sets		563
Appendix C: Weakening hypotheses		565
Appendix D: Ode to a galvanometer		575
Appendix E: The principle of the argument		577
Appendix F: Chase the constant		580
Appendix G: Are share prices in Brownian motion?		581
Index		585