

## TABLE OF CONTENTS

<u>Chapter 1:</u>	Historical Setting . . . . .	1
<u>Chapter 2:</u>	A Brief Introduction to Graph Theory . . . . .	5
	2-1. Definition of a graph . . . . .	5
	2-2. Variations of Graphs . . . . .	6
	2-3. Additional Definitions . . . . .	7
	2-4. Operations on Graphs . . . . .	9
	2-5. Problems . . . . .	12
<u>Chapter 3:</u>	The Automorphism Group of a Graph . . . . .	15
	3-1. Definitions . . . . .	15
	3-2. Operations on Permutation Groups . . . . .	17
	3-3. Computing Automorphism Groups of Graphs . . . . .	18
	3-4. Graphs with a Given Automorphism Group . . . . .	20
	3-5. Other Groups of a Graph . . . . .	20
	3-6. Problems . . . . .	21
<u>Chapter 4:</u>	The Cayley Color Graph of a Group Presentation . . . . .	23
	4-1. Definitions . . . . .	23
	4-2. Automorphisms . . . . .	26
	4-3. Properties . . . . .	28
	4-4. Products . . . . .	32
	4-5. Cayley Graphs . . . . .	35
	4-6. Problems . . . . .	36
<u>Chapter 5:</u>	An Introduction to Surface Topology . . . . .	39
	5-1. Definitions . . . . .	39
	5-2. Surfaces and Other 2-manifolds . . . . .	41
	5-3. The Characteristic of a Surface . . . . .	42
	5-4. Two Applications . . . . .	47
	5-5. Pseudosurfaces . . . . .	53
	5-6. Problems . . . . .	54
<u>Chapter 6:</u>	Imbedding Problems in Graph Theory . . . . .	57
	6-1. Answers to Some Imbedding Questions . . . . .	58
	6-2. Definition of "Imbedding" . . . . .	60
	6-3. The Genus of a Graph . . . . .	61
	6-4. The Maximum Genus of a Graph . . . . .	64
	6-5. Genus Formulae for Graphs . . . . .	67
	6-6. Edmonds' Permutation Technique . . . . .	70
	6-7. Imbedding Graphs on Pseudosurfaces . . . . .	73
	6-8. Other Topological Parameters for Graphs . . . . .	75
	6-9. Applications . . . . .	79
	6-10. Problems . . . . .	81

<u>Chapter 7:</u>	The Genus of a Group . . . . .	83
7-1.	Imbeddings of Cayley Color Graphs . . . . .	83
7-2.	Genus Formulae for Groups . . . . .	88
7-3.	Related Results . . . . .	95
7-4.	The Characteristic of a Group . . . . .	97
7-5.	Problems . . . . .	98
<u>Chapter 8:</u>	Map-coloring Problems . . . . .	101
8-1.	Definitions . . . . .	102
8-2.	The Four-color Conjecture . . . . .	102
8-3.	The Five-color Theorem . . . . .	106
8-4.	Other Map-coloring Problems; the Heawood Map-coloring Theorem . . . . .	107
8-5.	A Related Problem . . . . .	111
8-6.	A Four-color Theorem for the Torus . . . . .	114
8-7.	A Nine-color Theorem for the Torus and Klein Bottle . . . . .	117
8-8.	k-degenerate Graphs . . . . .	118
8-9.	Coloring Graphs on Pseudosurfaces . . . . .	120
8-10.	The Chromatic Number of Surfaces . . . . .	122
8-11.	Problems . . . . .	123
<u>Chapter 9:</u>	Quotient Graphs and Quotient Manifolds (and Quotient Groups!) . . . . .	125
9-1.	The Genus of $K_n$ . . . . .	125
9-2.	The Theory of Quotient Graphs and Quotient Manifolds, as Applied to $K_n$ . . . . .	127
9-3.	The Genus of $K_n$ (again) . . . . .	131
9-4.	Extending the Theory . . . . .	136
9-5.	The General Theory . . . . .	142
9-6.	Applications to Known Imbeddings . . . . .	149
9-7.	New Applications . . . . .	152
9-8.	Problems . . . . .	154
<u>Chapter 10:</u>	Voltage Graphs . . . . .	157
10-1.	Covering Spaces . . . . .	157
10-2.	Voltage Graphs . . . . .	160
10-3.	Examples . . . . .	164
10-4.	The Heawood Map-coloring Theorem (again) . . . . .	171
10-5.	Strong Tensor Products . . . . .	173
10-6.	Problems . . . . .	174
<u>Chapter 11:</u>	Nonorientable Graph Imbeddings . . . . .	177
11-1.	General Theory . . . . .	177
11-2.	Nonorientable Covering Spaces . . . . .	180
11-3.	Nonorientable Voltage Graph Imbeddings . . . . .	180
11-4.	Examples . . . . .	182
11-5.	The Heawood Map-coloring Theorem, Nonorientable Version . . . . .	184
11-6.	Other Results . . . . .	185
11-7.	Problems . . . . .	187

<u>Chapter 12:</u> Block Designs . . . . .	189
12-1. Balanced Incomplete Block Designs . . . . .	189
12-2. BIBDs and Graph Imbeddings . . . . .	190
12-3. Examples . . . . .	191
12-4. Strongly Regular Graphs . . . . .	192
12-5. Partially Balanced Incomplete Block Designs . . . . .	194
12-6. PBIBDs and Graph Imbeddings . . . . .	197
12-7. Examples . . . . .	198
12-8. Doubling a PBIBD . . . . .	201
12-9. Problems . . . . .	203
<u>Chapter 13:</u> Hypergraph Imbeddings . . . . .	205
13-1. Hypergraphs . . . . .	205
13-2. Associated Bipartite Graphs . . . . .	207
13-3. Imbedding Theory for Hypergraphs . . . . .	207
13-4. The Genus of a Hypergraph . . . . .	211
13-5. The Heawood Map-coloring Theorem, for Hypergraphs . . . . .	212
13-6. The Genus of a Block Design . . . . .	213
13-7. An Example . . . . .	215
13-8. Nonorientable Analogs . . . . .	217
13-9. Problems . . . . .	217
<u>Chapter 14:</u> Map Automorphism Groups . . . . .	219
14-1. Map Automorphisms . . . . .	219
14-2. Symmetrical Maps . . . . .	225
14-3. Cayley Maps . . . . .	229
14-4. Complete Maps . . . . .	233
14-5. Other Symmetrical Maps . . . . .	236
14-6. Self-complementary Graphs . . . . .	237
14-7. Self-dual Maps . . . . .	239
14-8. Paley Maps . . . . .	243
14-9. Problems . . . . .	254
<u>Chapter 15:</u> Change Ringing . . . . .	257
15-1. Definitions . . . . .	257
15-2. Notation . . . . .	260
15-3. General Theory . . . . .	262
15-4. Four-bell Extents (Minimus) . . . . .	266
15-5. Five-bell Extents (Doubles) . . . . .	270
15-6. A New Composition . . . . .	272
15-7. Problems . . . . .	276
<u>References</u> . . . . .	279
<u>Bibliography</u> . . . . .	303
<u>Index of Symbols</u> . . . . .	305
<u>Index of Definitions</u> . . . . .	309