
Contents

Preface	xi
Conventions	xvii
Part I. Algebras	
Chapter 1. Representations of Algebras	3
1.1. Algebras	3
1.2. Representations	24
1.3. Primitive Ideals	41
1.4. Semisimplicity	50
1.5. Characters	65
Chapter 2. Further Topics on Algebras	79
2.1. Projectives	79
2.2. Frobenius and Symmetric Algebras	96
Part II. Groups	
Chapter 3. Groups and Group Algebras	113
3.1. Generalities	113
3.2. First Examples	124
3.3. More Structure	131
3.4. Semisimple Group Algebras	143
3.5. Further Examples	150
3.6. Some Classical Theorems	159

3.7. Characters, Symmetric Polynomials, and Invariant Theory	170
3.8. Decomposing Tensor Powers	179
Chapter 4. Symmetric Groups	187
4.1. Gelfand-Zetlin Algebras	189
4.2. The Branching Graph	192
4.3. The Young Graph	197
4.4. Proof of the Graph Isomorphism Theorem	205
4.5. The Irreducible Representations	217
4.6. The Murnaghan-Nakayama Rule	222
4.7. Schur-Weyl Duality	235
Part III. Lie Algebras	
Chapter 5. Lie Algebras and Enveloping Algebras	245
5.1. Lie Algebra Basics	246
5.2. Types of Lie Algebras	253
5.3. Three Theorems about Linear Lie Algebras	257
5.4. Enveloping Algebras	266
5.5. Generalities on Representations of Lie Algebras	278
5.6. The Nullstellensatz for Enveloping Algebras	287
5.7. Representations of \mathfrak{sl}_2	300
Chapter 6. Semisimple Lie Algebras	315
6.1. Characterizations of Semisimplicity	316
6.2. Complete Reducibility	320
6.3. Cartan Subalgebras and the Root Space Decomposition	325
6.4. The Classical Lie Algebras	334
Chapter 7. Root Systems	341
7.1. Abstract Root Systems	342
7.2. Bases of a Root System	349
7.3. Classification	356
7.4. Lattices Associated to a Root System	361
Chapter 8. Representations of Semisimple Lie Algebras	373
8.1. Reminders	374
8.2. Finite-Dimensional Representations	377
8.3. Highest Weight Representations	379

8.4.	Finite-Dimensional Irreducible Representations	385
8.5.	The Representation Ring	390
8.6.	The Center of the Enveloping Algebra	393
8.7.	Weyl's Character Formula	408
8.8.	Schur Functors and Representations of $\mathfrak{sl}(V)$	418
Part IV. Hopf Algebras		
Chapter 9.	Coalgebras, Bialgebras, and Hopf Algebras	427
9.1.	Coalgebras	427
9.2.	Comodules	441
9.3.	Bialgebras and Hopf Algebras	447
Chapter 10.	Representations and Actions	465
10.1.	Representations of Hopf Algebras	466
10.2.	First Applications	476
10.3.	The Representation Ring of a Hopf Algebra	485
10.4.	Actions and Coactions of Hopf Algebras on Algebras	492
Chapter 11.	Affine Algebraic Groups	503
11.1.	Affine Group Schemes	503
11.2.	Affine Algebraic Groups	508
11.3.	Representations and Actions	512
11.4.	Linearity	515
11.5.	Irreducibility and Connectedness	520
11.6.	The Lie Algebra of an Affine Algebraic Group	526
11.7.	Algebraic Group Actions on Prime Spectra	530
Chapter 12.	Finite-Dimensional Hopf Algebras	541
12.1.	Frobenius Structure	541
12.2.	The Antipode	549
12.3.	Semisimplicity	552
12.4.	Divisibility Theorems	559
12.5.	Frobenius-Schur Indicators	567
Appendices		
Appendix A.	The Language of Categories and Functors	575
A.1.	Categories	575

A.2. Functors	578
A.3. Naturality	579
A.4. Adjointness	583
Appendix B. Background from Linear Algebra	587
B.1. Tensor Products	587
B.2. Hom- \otimes Relations	593
B.3. Vector Spaces	594
Appendix C. Some Commutative Algebra	599
C.1. The Nullstellensatz	599
C.2. The Generic Flatness Lemma	601
C.3. The Zariski Topology on a Vector Space	602
Appendix D. The Diamond Lemma	605
D.1. The Goal	605
D.2. The Method	606
D.3. First Applications	608
D.4. A Simplification	611
D.5. The Poincaré-Birkhoff-Witt Theorem	612
Appendix E. The Symmetric Ring of Quotients	615
E.1. Definition and Basic Properties	615
E.2. The Extended Center	617
E.3. Comparison with Other Rings of Quotients	619
Bibliography	623
Subject Index	633
Index of Names	645
Notation	649