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**Novel Algorithms
Towards Systematic,
Generalized And
Compact Recursive
Generation
And Computation Of
The Complete Robot
Dynamics With
Realisation Aspects
In CAD-Systems**

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