

M. Sc. Xiaobo Wang, Erlangen

# **Modal Analysis of Large Interconnected Power Systems**

Reihe **6**: Energietechnik

Nr. **380**

## Content

Nomenclature .....	VIII
Abbreviations .....	X
Abstract .....	XI
<b>1. Introduction .....</b>	<b>1</b>
1.1 Power System Stability and Methods of Analysis .....	1
1.2 Power System Small Signal Stability and Frequency Domain Methods .....	2
1.3 Comparison Between Simulation Method and Modal Analysis .....	3
1.4 Challenges in Development of Modal Analysis .....	6
1.5 Initiatives and Scope of the Work .....	6
1.6 Outline of This Publication .....	7
<b>2. Fundamentals of Linear Systems and Eigenvalue Theory .....</b>	<b>8</b>
2.1 State Space Representation of Linear Dynamic Systems .....	8
2.2 Transfer Function .....	9
2.3 Mathematical Definitions of Eigenvalues and Eigenvectors .....	10
2.4 Time Response of Linear Dynamic Systems .....	12
2.5 Physical Meanings of Eigenvalues .....	15
2.6 Physical Meanings of Eigenvectors .....	17
2.7 Extended Concepts of State Equations and Eigensystems .....	18
<b>3. Linear Representation of Dynamic Power Systems .....</b>	<b>21</b>
3.1 Linearization .....	21
3.2 Assembly of Implicit State Equation .....	24
3.3 Features of Implicit State Equation .....	28
<b>4. Linear Models of Power System Components .....</b>	<b>31</b>
4.1 Overview .....	31
4.2 Synchronous Machine .....	33
4.3 Asynchronous Machine .....	47
4.4 Passive Elements of Transmission Network .....	48
4.5 Controlled Power Source .....	51
4.6 Controlled Voltage Source .....	53
4.7 Controlled Current Source .....	54
4.8 Controlled Impedance Branch .....	55

4.9	Controlled Tap Changer Branch .....	58
4.10	User Defined Controller .....	60
<b>5.</b>	<b>Full Eigenvalue Solution .....</b>	<b>62</b>
5.1	QR Transformation .....	62
5.2	Calculation Procedure .....	64
5.3	Eigenvalue Results with a Benchmark System .....	67
<b>6.</b>	<b>Partial Eigenvalue Solution .....</b>	<b>69</b>
6.1	Survey of Partial Eigenvalue Solution Methods .....	70
6.2	Spectral Transformation .....	73
6.3	Dominant Poles of a Transfer Function .....	75
6.4	Exploiting Sparse Properties of Implicit State Matrices .....	76
6.5	Implicit Inverse Iteration .....	79
6.6	Rayleigh Quotient Iteration .....	83
6.7	Dominant Pole Iteration .....	85
<b>7.</b>	<b>Modal Analysis .....</b>	<b>89</b>
7.1	Mode Overview .....	89
7.2	Right Eigenvectors and Mode Observability (Mode Shape) .....	95
7.3	Left Eigenvectors and Mode Controllability .....	101
7.4	Participation Factors .....	104
7.5	Transfer Function Residues and Controller Siting .....	108
7.6	Mode Sensitivities to Parameter Changes .....	120
7.7	Frequency Response .....	122
7.8	Linear Time Response Through Impulse/Step Disturbances .....	126
<b>8.</b>	<b>Applications on Large Interconnected Power Systems .....</b>	<b>129</b>
8.1	General Discussions on Modes of Power Systems .....	130
8.2	General Discussions on Local and Interarea Modes .....	132
8.3	Study of <i>Power System 1</i> , a Longitudinal National Grid .....	136
8.4	Study of <i>Power System 2</i> , a Multi-National Power Pool .....	146
8.5	Study of <i>Power System 3</i> , a Very Large Power System .....	155
<b>9.</b>	<b>Summary .....</b>	<b>158</b>
9.1	Conclusions .....	158
9.2	Outlook .....	159

---

<b>Appendix A. Miscellaneous</b> .....	<b>160</b>
A.1 Explanation of Time Constant of Amplitude Decay $\tau_\sigma$ .....	160
A.2 Explanation of Damping Ratio $\zeta$ .....	160
A.3 Different Meanings of “a Dominant Mode” .....	162
A.4 Derivation of Equations for Eigenvalue Sensitivity $\partial s/\partial q$ .....	163
A.5 Swing Frequency of a Classical Generator Connected to an Infinite Bus .....	164
<b>Appendix B. Linear Interpretation of Control Simulation Language</b> .....	<b>165</b>
B.1 Importing System Variables As Input Signals .....	165
B.2 Dynamic Elements .....	166
B.3 Arithmetic Operations .....	168
B.4 Mathematical Functions .....	169
B.5 Other Elements .....	173
<b>References</b> .....	<b>174</b>