Table of contents

Chap. 1 Existence and main properties of **W**

1.0 Introduction 1

1.1 Existence of **W** and first properties 1

1.1.1 A few more notations 1

1.1.2 A Feynman-Kac penalisation result 2

1.1.3 Definition of **W** 3

1.1.4 Study of the canonical process under $W_\infty^{(\lambda\delta_0)}$ 6

1.1.5 Some remarkable properties of **W** 7

1.1.6 Another approach to Theorem 1.1.6 14

1.1.7 Relation between **W** and other penalisations (than the Feynman-Kac ones) 16

1.2 W-Brownian martingales associated to **W** 20

1.2.1 Definition of the martingales $\big(M_t(F),\ t \geq 0\big)$ 20

1.2.2 Examples of martingales $\big(M_t(F),\ t \geq 0\big)$ 23

1.2.3 A decomposition Theorem for positive Brownian supermartingales 32

1.2.4 A decomposition result for the martingale $\big(M_t(F),\ t \geq 0\big)$ 36

1.2.5 A penalisation Theorem, for functionals in class $\mathcal{C}$ 48

1.2.6 Some other results about the martingales $(M_t(F),\ t \geq 0)$ 54

1.3 Invariant measures related to $\mathbf{W}_x$ and $\mathbf{\Lambda}_x$ 59

1.3.1 The process $(\mathcal{X}_t,\ t \geq 0)$ 59

1.3.2 The measure $\mathbf{\Lambda}_x$ 61

1.3.3 Invariant measures for the process $\big((X_t, L_t^\bullet),\ t \geq 0\big)$ 62

1.3.4 Invariant measures for the process $(L_t^{X_t - \bullet},\ t \geq 0)$ 69

Chap. 2 Existence and properties of $\mathbf{W}^{(2)}$

2.1 Existence of $\mathbf{W}^{(2)}$ 73

2.1.1 Notation and Feynman-Kac penalisations in two dimensions 73

2.1.2 Existence of the measure $\mathbf{W}^{(2)}$ 74

2.2 Properties of $\mathbf{W}^{(2)}$ 75

2.2.1 Some notation 75

2.2.2 Description of the canonical process under $W_\infty^{(2,q_0)}$ 77

2.2.3 Another description of the measure $\mathbf{W}^{(2)}$ 79

2.3 Study of the winding process under $\mathbf{W}^{(2)}$ 81

2.3.1 Spitzer's Theorem 81

2.3.2 An analogue of Spitzer's Theorem 81

2.4 $W^{(2)}$-martingales associated to $\mathbf{W}^{(2)}$ 84

2.4.1 Definition of $\left(M_t^{(2)}(F),\ t \geq 0\right)$ 84

2.4.2 A decomposition Theorem for positive $W^{(2)}$-supermartingales 85

2.4.3 A decomposition Theorem of the martingales $\left(M_t^{(2)}(F),\ t \geq 0\right)$ 86

Chap. 3 The analogue of the measure $\mathbf{W}$ for a class of linear diffusions

3.1 Main hypotheses and notations 89

3.1.1 Our framework is that of Salminen-Vallois-Yor 89

3.1.2 The semi-group of $(X_t,\ t \geq 0)$ 89

3.1.3 The local time process 89

3.1.4 The process X conditioned not to vanish 90

3.1.5 A useful Proposition 90

3.2 The σ-finite measure $\mathbf{W}^*$ 91

3.2.1 Definition of $\mathbf{W}^*$ 91

3.2.2 Some properties of $\mathbf{W}^*$ 92

3.2.3 Relation between the measure $\mathbf{W}^*$ and penalisations 96

3.3 The example of Bessel processes with dimension d $(0 < d < 2)$ 97

3.3.1 Transcription of our notation in the context of Bessel processes 98

3.3.2 The measure $\mathbf{W}^{(-\alpha)}$ 98

3.3.3 Relations between $\mathbf{W}^{(-\alpha)}$ $\left(d = 2(1-\alpha)\right)$ and Feynman-Kac penalisations 99

3.4 Another description of $\mathbf{W}^{(-\alpha)}$ (and of $\mathbf{W}_g^*$) 101

3.5 Penalisations of α-stable symmetric Lévy process $(1 < \alpha \leq 2)$ 103

3.5.1 Notation and classical results 104

3.5.2 Definition of the σ-finite measure $\mathbf{P}$ 105

3.5.3 The martingales $\left(M_t(F),\ t \geq 0\right)$ associated with $\mathbf{P}$ 106

3.5.4 Relations between $\mathbf{P}$ and penalisations 107

Chap. 4 An analogue of the measure $\mathbf{W}$ for discrete Markov chains

4.0 Introduction 109

4.1 Construction of the σ-finite measures $(\mathbb{Q}_x,\ x \in E)$ 109

4.1.1 Notation and hypothesis 109

4.1.2 A family of new measures 109

4.1.3 Definition of the measures $(\mathbb{Q}_x,\ x \in E)$ 111

4.2 Some properties of $(\mathbb{Q}_x,\ x \in E)$ 114

4.2.1 Martingales associated with $(\mathbb{Q}_x,\ x \in E)$ 114

4.2.2 Properties of the canonical process under $(\mathbb{Q}_x,\ x \in E)$ 115

4.2.3 Dependence of $\mathbb{Q}_x$ on x_0 117

4.2.4 Dependence of $\mathbb{Q}_x$ on ϕ 120

4.3 Some examples 122

4.3.1 The standard random walk 122

4.3.2 The "bang-bang random walk" 123

4.3.3 The random walk on a tree 124

4.3.4 Some more general conditions for the existence of ϕ 125

4.3.5 The standard random walk on $\mathbb{Z}^2$ 126