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**Derivation and Finite
Element Implementation of
Constitutive Material Laws
for Multiphase Composites
Based on Mori-Tanaka
Approaches**

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