

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

*Combinatorics, Words and
Symbolic Dynamics*

Edited by

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