

# Abstract Algebraic Logic

An Introductory Textbook

Josep Maria Font

---

# Short contents

---

<b>Short contents</b>	<b>vii</b>
<b>Detailed contents</b>	<b>ix</b>
<b>A letter to the reader</b>	<b>xv</b>
<b>Introduction and Reading Guide</b>	<b>xix</b>
<b>1 Mathematical and logical preliminaries</b>	<b>1</b>
1.1 Sets, languages, algebras . . . . .	1
1.2 Sentential logics . . . . .	12
1.3 Closure operators and closure systems: the basics . . . . .	32
1.4 Finitarity and structurality . . . . .	44
1.5 More on closure operators and closure systems . . . . .	54
1.6 Consequences associated with a class of algebras . . . . .	59
<b>2 The first steps in the algebraic study of a logic</b>	<b>71</b>
2.1 From two-valued truth tables to Boolean algebras: the Lindenbaum-Tarski process for classical logic . . . . .	71
2.2 Implicative logics . . . . .	76
2.3 Filters . . . . .	88
2.4 Extensions of the Lindenbaum-Tarski process . . . . .	98
2.5 Two digressions on first-order logic . . . . .	102
<b>3 The semantics of algebras</b>	<b>107</b>
3.1 Transformers, algebraic semantics, and assertional logics . . . . .	108
3.2 Algebraizable logics . . . . .	115
3.3 A syntactic characterization, and the Lindenbaum-Tarski process again . . . . .	123
3.4 More examples, and special kinds of algebraizable logics . . . . .	129
3.5 The Isomorphism Theorems . . . . .	145
3.6 Bridge theorems and transfer theorems . . . . .	159
3.7 Generalizations and abstractions of algebraizability . . . . .	176

<b>4</b>	<b>The semantics of matrices</b>	<b>183</b>
4.1	Logical matrices: basic concepts . . . . .	183
4.2	The Leibniz operator . . . . .	194
4.3	Reduced models and Leibniz-reduced algebras . . . . .	203
4.4	Applications to algebraizable logics . . . . .	214
4.5	Matrices as relational structures . . . . .	220
<b>5</b>	<b>The semantics of generalized matrices</b>	<b>235</b>
5.1	Generalized matrices: basic concepts . . . . .	236
5.2	Basic full generalized models, Tarski-style conditions and transfer theorems . . . . .	244
5.3	The Tarski operator and the Suszko operator . . . . .	254
5.4	The algebraic counterpart of a logic . . . . .	270
5.5	Full generalized models . . . . .	285
5.6	Generalized matrices as models of Gentzen systems . . . . .	304
<b>6</b>	<b>Introduction to the Leibniz hierarchy</b>	<b>317</b>
6.1	Overview . . . . .	317
6.2	Protoalgebraic logics . . . . .	322
6.3	Definability of equivalence (protoalgebraic and equivalential logics) . . . . .	352
6.4	Definability of truth (truth-equational, assertional and weakly algebraizable logics) . . . . .	371
6.5	Algebraizable logics revisited . . . . .	400
<b>7</b>	<b>Introduction to the Frege hierarchy</b>	<b>413</b>
7.1	Overview . . . . .	413
7.2	Selfextensional and fully selfextensional logics . . . . .	419
7.3	Fregean and fully Fregean logics . . . . .	449
	<b>Summary of properties of particular logics</b>	<b>471</b>
	<b>Bibliography</b>	<b>485</b>
	<b>Indices</b>	<b>497</b>