

Contents

<i>Preface to the Second Edition</i>	vii
<i>Preface to the First Edition (2001)</i>	ix
1. Quadratic Equations	1
1.1 Babylonian algebra	2
1.2 Greek algebra	5
1.3 Arabic algebra	9
2. Cubic Equations	13
2.1 Priority disputes on the solution of cubic equations	13
2.2 Cardano's formula	15
2.3 Developments arising from Cardano's formula	16
3. Quartic Equations	21
3.1 The unnaturalness of quartic equations	21
3.2 Ferrari's method	22
4. The Creation of Polynomials	25
4.1 The rise of symbolic algebra	25
4.1.1 L'Arithmetique	26
4.1.2 In Artem Analyticem Isagoge	29
4.2 Relations between roots and coefficients	30
5. A Modern Approach to Polynomials	41
5.1 Definitions	41
5.2 Euclidean division	43

5.3	Irreducible polynomials	48
5.4	Roots	51
5.5	Multiple roots and derivatives	53
5.6	Common roots of two polynomials	56
	Appendix: Decomposition of rational functions into sums of partial fractions	60
6.	Alternative Methods for Cubic and Quartic Equations	63
6.1	Viète on cubic equations	63
	6.1.1 Trigonometric solution for the irreducible case . .	63
	6.1.2 Algebraic solution for the general case	64
6.2	Descartes on quartic equations	66
6.3	Rational solutions for equations with rational coefficients .	67
6.4	Tschirnhaus' method	68
7.	Roots of Unity	73
7.1	The origins of de Moivre's formula	73
7.2	The roots of unity	80
7.3	Primitive roots and cyclotomic polynomials	85
	Appendix: Leibniz and Newton on the summation of series . . .	89
	Exercises	90
8.	Symmetric Functions	93
8.1	Waring's method	96
8.2	The discriminant	101
	Appendix: Euler's summation of the series of reciprocals of perfect squares	105
	Exercises	107
9.	The Fundamental Theorem of Algebra	109
9.1	Girard's theorem	110
9.2	Proof of the fundamental theorem	113
10.	Lagrange	117
10.1	The theory of equations comes of age	117
10.2	Lagrange's observations on previously known methods . .	121
10.3	First results of group theory and Galois theory	131
	Exercises	142

11. Vandermonde	143
11.1 The solution of general equations	144
11.2 Cyclotomic equations	148
Exercises	154
12. Gauss on Cyclotomic Equations	155
12.1 Number-theoretic preliminaries	156
12.2 Irreducibility of the cyclotomic polynomials of prime index	162
12.3 The periods of cyclotomic equations	169
12.4 Solvability by radicals	178
12.5 Irreducibility of the cyclotomic polynomials	182
Appendix: Ruler and compass construction of regular polygons	185
Exercises	192
13. Ruffini and Abel on General Equations	193
13.1 Radical extensions	195
13.2 Abel's theorem on natural irrationalities	203
13.3 Proof of the unsolvability of general equations of degree higher than 4	209
Exercises	211
14. Galois	215
14.1 Arrangements and permutations	220
14.2 The Galois group of an equation	225
14.3 The Galois group under base field extension	236
14.4 Solvability by radicals	246
14.5 Applications	256
14.5.1 Irreducible equations of prime degree	256
14.5.2 Abelian equations	265
Exercises	268
15. Epilogue	271
Appendix 1: The fundamental theorem of Galois theory	274
Appendix 2: Galois theory <i>à la</i> Grothendieck	283
Étale algebras	283
Galois algebras	285
Galois groups	287
Exercises	290

<i>Selected Solutions</i>	291
<i>Bibliography</i>	299
<i>Index</i>	305