

Contents

Preface	vii
Chapter 1. Basic Definitions and Examples	1
1.1. Rooted tree X^* and its boundary X^ω	1
1.2. Groups acting on rooted trees	2
1.3. Automata	3
1.4. Wreath products	9
1.5. Self-similar actions	10
1.6. The Grigorchuk group	12
1.7. The adding machine and self-similar actions of \mathbb{Z}^n	16
1.8. Branch groups	17
1.9. Other examples	21
1.10. Bi-reversible automata and free groups	23
Chapter 2. Algebraic Theory	31
2.1. Permutational bimodules	31
2.2. Bases of a covering bimodule and wreath recursions	32
2.3. Tensor products and self-similar actions	33
2.4. The left G -space $\mathfrak{M}^{\otimes\omega}$	36
2.5. Virtual endomorphisms	37
2.6. The linear recursion	40
2.7. Invariant subgroups and the kernel of a self-similar action	41
2.8. Recurrent actions	44
2.9. Example: free abelian groups	46
2.10. Rigidity	50
2.11. Contracting actions	57
2.12. Finite-state actions of \mathbb{Z}^n	62
2.13. Defining relations and word problem	64
Chapter 3. Limit Spaces	71
3.1. Introduction	71
3.2. The limit G -space \mathcal{X}_G	73
3.3. Digit tiles	78
3.4. Axiomatic description of \mathcal{X}_G	82
3.5. Connectedness of \mathcal{X}_G	91
3.6. The limit space \mathcal{J}_G	92
3.7. Limit spaces of self-similar subgroups	96
3.8. The limit space \mathcal{J}_G as a hyperbolic boundary	97
3.9. Groups of bounded automata	102
3.10. One-dimensional subdivision rules	110

3.11. Uniqueness of the limit space	113
Chapter 4. Orbispaces	117
4.1. Pseudogroups and étale groupoids	117
4.2. Orbispaces	119
4.3. Open sub-orbispaces and coverings	122
4.4. Coverings and skew-products	124
4.5. Partial self-coverings	127
4.6. The limit orbispace \mathcal{J}_G	128
4.7. Paths in an orbispace	131
Chapter 5. Iterated Monodromy Groups	137
5.1. Definition of iterated monodromy groups	137
5.2. Standard self-similar actions of $\text{IMG}(p)$ on X^*	142
5.3. Iterated monodromy groups of limit dynamical systems	146
5.4. Length structures and expanding maps	148
5.5. Limit spaces of iterated monodromy groups	150
5.6. Iterated monodromy group of a pull-back	154
5.7. The limit solenoid and inverse limits of self-coverings	156
Chapter 6. Examples and Applications	161
6.1. Expanding self-coverings of orbifolds	161
6.2. Limit spaces of free Abelian groups	165
6.3. Examples of self-coverings of orbifolds	169
6.4. Rational functions	174
6.5. Combinatorial equivalence and Thurston's Theorem	176
6.6. "Twisted rabbit" question of J. Hubbard	179
6.7. Abstract kneading automata	185
6.8. Topological polynomials and critical portraits	190
6.9. Iterated monodromy groups of complex polynomials	193
6.10. Polynomials from kneading automata	196
6.11. Quadratic polynomials	203
6.12. Examples of iterated monodromy groups of polynomials	208
6.13. Matings	215
Bibliography	223
Index	229