

TABLE OF CONTENTS

| | PAGE |
|--|------|
| INTRODUCTION | 1 |
| CHAPTER | |
| 1. THE PROPOSITIONAL CALCULUS | 12 |
| 1. Propositional Connectives. Truth Tables | 12 |
| 2. Tautologies | 17 |
| 3. Adequate Sets of Connectives | 24 |
| 4. An Axiom System for the Propositional Calculus | 29 |
| 5. Independence. Many-Valued Logics | 38 |
| 6. Other Axiomatizations | 40 |
| 2. QUANTIFICATION THEORY | 45 |
| 1. Quantifiers | 45 |
| 2. Interpretations. Satisfiability and Truth. Models | 49 |
| 3. First-Order Theories | 56 |
| 4. Properties of First-Order Theories | 59 |
| 5. Completeness Theorems | 62 |
| 6. Some Additional Metatheorems | 70 |
| 7. Rule C | 73 |
| 8. First-Order Theories with Equality | 75 |
| 9. Definitions of New Function Letters and Individual Constants | 82 |
| 10. Prenex Normal Forms | 85 |
| 11. Isomorphism of Interpretations. Categoricity of Theories | 90 |
| 12. Generalized First-Order Theories. Completeness and Decidability | 92 |
| 3. FORMAL NUMBER THEORY | 102 |
| 1. An Axiom System | 102 |
| 2. Number-Theoretic Functions and Relations | 117 |
| 3. Primitive Recursive and Recursive Functions | 120 |
| 4. Arithmetization. Gödel Numbers | 135 |
| 5. Gödel's Theorem for S | 142 |
| 6. Recursive Undecidability. Tarski's Theorem. Robin- son's System | 150 |

| CHAPTER | PAGE |
|---|------|
| 4. AXIOMATIC SET THEORY | 159 |
| 1. An Axiom System | 159 |
| 2. Ordinal Numbers | 170 |
| 3. Equinumerosity. Finite and Denumerable Sets | 180 |
| 4. Hartogs' Theorem. Initial Ordinals. Ordinal Arithmetic | 187 |
| 5. The Axiom of Choice. The Axiom of Restriction | 197 |
| 5. EFFECTIVE COMPUTABILITY | 207 |
| 1. Markov Algorithms | 207 |
| 2. Turing Algorithms | 229 |
| 3. Herbrand-Gödel Computability. Recursively Enumer- able Sets | 238 |
| 4. Undecidable Problems | 254 |
| APPENDIX: A CONSISTENCY PROOF FOR FORMAL NUMBER THEORY | 258 |
| BIBLIOGRAPHY | 272 |
| INDEX | 291 |