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PART I—FUNDAMENTAL METHODS

1. THE CALCULATION OF FUNCTIONS 5

Wherein the student is introduced to the table-less computer that must evaluate its transcendental functions by divers means. Among others, the power series, continued fraction, and rational function approximations for the arctangent are displayed. Infinite products and asymptotic series appear briefly. We recommend recurrence relations for the evaluation of orthogonal series. In our final example we approximate, at length, a function defined by a quadrature over an infinite interval.

2. ROOTS OF TRANSCENDENTAL EQUATIONS 41

False Position, Newton's method and more specialized techniques introduce the basic ideas of iteration. Double root difficulties and elementary rate-of-convergence estimations are hidden among more important topics such as the need for good starting values and the geometric ideas that lead to workable algorithms. Divergence and significant figure loss make their debut. As a summary example, we analyze the solutions of $(px + q)e^x = rx + s$ in considerable detail.

3. INTERPOLATION—AND ALL THAT! 89

Being the irreducible minimum about interpolation that an engineer needs to know. Bessel's, Everett's and Aitken's methods are exhibited and recommended. Lagrangian interpolation is praised for analytic utility and beauty but deplored for numerical practice. A short chapter.

4. QUADRATURE 100

Simpson, Gauss, and the method of undetermined coefficients are recommended. Extrapolation to the limit à la Romberg appears briefly. Considerable effort goes into dealing with infinite ranges of integration. Simple singularities of the integrand enter, menacingly, just before the end.

**5. ORDINARY DIFFERENTIAL EQUATIONS—
INITIAL CONDITIONS 129**

We introduce extrapolation from initial conditions mostly in the context of predictor-corrector algorithms. Runge-Kutta is mentioned only as a good way to start. The philosophy of Bulirsch and Stoer is exhibited. We discuss the stability of Milne's method before passing to "stiff" equations and other forms of trouble that, if ignored, can wreck *any* standard integration scheme.

**6. ORDINARY DIFFERENTIAL EQUATIONS—
BOUNDARY CONDITIONS 157**

Algebraic replacement of the two-point boundary value problem on a grid quickly leads to the solution of tridiagonal algebraic systems. Infinite ranges reappear, and a sample problem is

solved. We then solve a slightly nonlinear differential equation — and the student finally learns how nasty the world really is. We offer him a Hobson's choice between linearized algebraic iteration and a retreat to initial value integration techniques (iterated).

7. STRATEGY VERSUS TACTICS— ROOTS OF POLYNOMIALS 178

A discussion of some of the better methods, emphasizing both the need for an overall plan and the futility of trying to solve everybody's polynomial with a single system. We prefer Newton's method for refining isolated real roots, isolated complex roots, and isolated quadratic factors. Stepping searches, division of one polynomial by another, the Forward-Backward division algorithm, Lin's method for quadratic factors of difficult quartics, and Laguerre's method receive various amounts of attention. Root-squaring is not recommended. After discussing the *desiderata* for a public root-finding computer package, we finally raise the spectre of Wilkinson's pathologic polynomial.

8. EIGENVALUES I 204

An introductory discussion of the power method for finding the extreme eigenvalues of symmetric matrices. Three vibrating masses briefly introduce the subject. Shifting an eigenvalue into prominence and the art of running orthogonal to an extreme eigenvector are treated. A final section points to the unsymmetric eigenproblem but does not discuss it in realistic detail. In this chapter a brief exposure to the ideas and typography of vectors and matrices is helpful, although a lack of it should not stymie the student.

9. FOURIER SERIES 221

We stress the practical evaluation of Fourier series by recurrence relations, especially the finite Fourier series with its summation orthogonalities. Prior removal of singularities, suppression of the Gibb's phenomenon, and the role of various kinds of symmetry appear. We conclude with an extension of the recurrence scheme to polynomials orthogonal over a finite irregular set of points and a mere mention of the exponential formalism of the finite Fourier transform.

INTERLUDE—WHAT NOT TO COMPUTE—
A BRIEF CATHARTIC ESSAY

PART II—DOUBLE TROUBLE

10. THE EVALUATION OF INTEGRALS 261

A detailed examination of two definite integrals as functions of their parameters. We find representations for them as power series, differential equations, asymptotic series, and rational functions. Our emphasis falls on assessing and removing the singularity—and the need for the economization procedures of Chapter 12.

**11. POWER SERIES, CONTINUED FRACTIONS,
AND RATIONAL APPROXIMATIONS — 279**

A convenient cookbook of algorithms for transforming infinite series into continued fractions of several kinds, and vice versa—not to mention rational functions. The important quotient-difference algorithm appears.

12. ECONOMIZATION OF APPROXIMATIONS 289

A discussion of the ideas underlying economization of power series and rational functions, especially via Chebyshev polynomials and minimax criteria. Remes's second algorithm appears. We also actually compute some approximations—using both direct economization and Maehly's method for fitting the discrepancy, or "tail," of the rational function.

13. EIGENVALUES II—ROTATIONAL METHODS 316

A Wilkinsonian treatment of rotational methods for symmetric and unsymmetric matrices. Similarity and orthogonal transformations to produce tridiagonal and Hessenberg forms introduce the reductions of Jacobi, Givens, and Householder. We discuss root finding from characteristic polynomials expressed as a tridiagonal matrix. Sturmian sequences and some forms of degeneracy appear. The LR , QR , and Cholesky separations for nearly simultaneous liberation of all eigenvalues are described.

Work loads required by the commoner matrix configurations lead to general recommendations. Finally, we talk about finding *eigenvectors*—something not everyone really wants.

14. ROOTS OF EQUATIONS—AGAIN 361

A return to the subject of Chapter 2, but this time in several dimensions and for a hopefully more experienced clientel. We point out more difficulties than we solve. The emphasis is on describing several reasonable strategies plus a plea that you seek strategies suitable to the particular problem. The chapter closes with three detailed examples of iterative procedures suggested by their geometries.

15. THE CARE AND TREATMENT OF SINGULARITIES 410

How to seek and destroy singularities in integrands and integrals. The use of trigonometric and elliptic function transformations. Logarithmic singularities. By way of example, five nasty integrals are tamed. Above all, a reiterated warning against wishful avoidance of singularities.

16. INSTABILITY IN EXTRAPOLATION 431

A description of numerical cancer: the insidious erosion of otherwise useful algorithms by exponentially amplified errors. We find the evil in recurrence relations, ordinary differential equations integrated from initial conditions, and in parabolic partial differential equations. We stress the distinction between finite difference replacements that are unstable and those that are merely imprecise.

17. MINIMUM METHODS 448

An exposition of some of the more effective ways to find minima in several dimensions, with a plea that other strategies for solving your problem be tried first. Stepping searches, ray minimum methods, and ellipsoidal center seekers appear. The influence of both the availability and pertinence of derivative information is stressed.

18. LAPLACE'S EQUATION—AN OVERVIEW 477

The two-dimensional Dirichlet problem introduces algebraic replacement on a grid. We soon pass to strategies for solving fringed tridiagonal linear algebraic systems, including the method of alternating directions. Boundary condition replacement on irregular boundaries are glanced at and found possible, though messy. Finally, we consider the finite parallel plate condenser by four unrelated methods. Our interest centers on the problems caused by the incompleteness of the boundary and the inconspicuousness of the singularities.

19. NETWORK PROBLEMS 499

A brief postlude on a less classical computational topic. We examine traffic through Baltimore, topological sorting of nodes in an ordered network, minimum tree construction in Alabama, and flow through a network of pipes. The algorithms are by Ford and Fulkerson, Kahn, and others.

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