CONTENTS

PREFACE vii

1 SOLUTION OF EQUATIONS BY FIXED-POINT ITERATION 1

1-1 Introduction 2
1-2 Example 2
1-3 Principal Algorithm 3
1-4 Review of Basic Concepts 7
1-5 Principal Theorems 13
1-6 Newton-Raphson Iteration 19
1-7 Graphical Analysis 22
1-8 Error Bounds 26

2 MATRIX COMPUTATIONS AND SOLUTION OF LINEAR EQUATIONS 32

2-1 Introduction 33
2-2 Matrices 34
2-3 Upper-Triangular Systems 43
2-4 Reduction of a Linear System to Upper-Triangular Form 47
2-5 The Inverse of a Square Matrix 53
2-6 A Better Algorithm for Inverting a Matrix 58
2-7 Error Aspects 61
2-8 Estimation of Computation Times 64

3 ITERATIVE SOLUTION OF SYSTEMS OF EQUATIONS 65

3-1 Introduction 66
3-2 Functions of n Variables 73
3-3 Iteration for Linear Systems 83
3-4 Nonlinear Systems of Equations 93

4 POLYNOMINALS, TAYLOR'S SERIES, AND INTERPOLATION THEORY 108

4-1 Introduction 109
4-2 Evaluation of a Polynomial 110
4-3 The Taylor Expansion 117
4-4 Linear Interpolation 123
4-5 Lagrange Interpolation 127
4-6 Iterated Interpolation 135

5 ERRORS AND FLOATING-POINT ARITHMETIC 141

5-1 Introduction 142
5-2 Errors 142
5-3 Floating-point Arithmetic 150
5-4 Examples from Newton-Raphson Iteration 155
5-5 Linear-interpolation Example 158
5-6 Inversion of an Ill-conditioned Matrix 159
5-7 Examples from Taylor's Series 162
5-8 Examples of an Infinite Sum 163

6 NUMERICAL DIFFERENTIATION AND INTEGRATION 166

6-1 Introduction 167
6-2 Numerical Differentiation 167
6-3 Numerical Integration 177
6-4 Formulas for Equally Spaced Points 179
6-5 Remainder Terms 182
6-6 Integration over Large Intervals 189
6-7 Richardson's Extrapolation 194
6-8 Romberg Integration 199
6-9 Multiple Integrals 206

7 INTRODUCTION TO THE NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS 208

7-1 Introduction 209
7-2 Existence Theorems for a Single ODE 212
7-3 Systems of First-order Equations 217
7-4 Euler's Method 227

7-5 Heun's Method 236
7-6 Error-analysis Aspects 241

8 NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS 246

8-1 Introduction 247
8-2 Taylor's-Series Procedures 247
8-3 Runge-Kutta Methods 257
8-4 Predictor-Corrector Methods 267

REFERENCES 279
ANSWERS TO SELECTED EXERCISES 281
INDEX 293