

# Contents

Preface .....	v
Notation and Symbols .....	xi
Introduction .....	1
1. Models .....	1
2. The Probability Space .....	3
3. Independence and Conditional Probabilities .....	5
4. Random Variables .....	6
5. Expectation, Variance, and Moments .....	8
6. Joint Distributions and Independence .....	9
7. Sums of Random Variables, Covariance, and Correlation .....	10
8. Limit Theorems .....	12
9. Stochastic Processes .....	13
10. The Contents of the Book .....	13
I. Multivariate Random Variables .....	17
1. Introduction .....	17
2. Functions of Random Variables .....	21
3. Problems .....	27
II. Conditioning .....	32
1. Conditional Distributions .....	32
2. Conditional Expectation and Conditional Variance .....	34
3. Distributions with Random Parameters .....	40
4. The Bayesian Approach .....	45
5. Regression and Prediction .....	49
6. Martingales .....	53
7. Problems .....	55
III. Transforms .....	60
1. Introduction .....	60
2. The Probability Generating Function .....	62

3. The Moment Generating Function .....	66
4. The Characteristic Function .....	73
5. Sums of a Random Number of Random Variables .....	80
6. Branching Processes .....	87
7. Distributions with Random Parameters .....	94
8. Problems .....	96
IV. Order Statistics .....	102
1. One-Dimensional Results .....	102
2. The Joint Distribution of the Extremes .....	107
3. The Joint Distribution of the Order Statistic .....	111
4. Problems .....	115
V. The Multivariate Normal Distribution .....	119
1. Preliminaries from Linear Algebra .....	119
2. The Covariance Matrix .....	122
3. A First Definition .....	123
4. The Characteristic Function: Another Definition .....	125
5. The Density: A Third Definition .....	127
6. Conditional Distributions .....	129
7. Independence .....	133
8. Linear Transformations .....	135
9. Quadratic Forms and Cochran's Theorem .....	139
10. Problems .....	143
VI. Convergence .....	149
1. Definitions .....	149
2. Uniqueness .....	152
3. Relations Between the Convergence Concepts .....	155
4. The Borel-Cantelli Lemmas and Complete Convergence ..	163
5. Convergence via Transforms .....	169
6. The Law of Large Numbers and the Central Limit Theorem .....	172
7. Convergence of Sums of Sequences of Random Variables ..	177
8. Problems .....	186
VII. The Poisson Process .....	195
1. Introduction and Definitions .....	195
2. Restarted Poisson Processes .....	207
3. Conditioning on the Number of Occurrences in an Interval .....	215
4. Conditioning on Occurrence Times .....	220
5. Several Independent Poisson Processes .....	222

6. Thinning of Poisson Processes .....	231
7. The Compound Poisson Process .....	236
8. Some Further Generalizations and Remarks .....	238
9. Problems .....	246
Appendixes .....	253
1. Suggestions for Further Reading .....	253
2. Some Distributions and Their Characteristics .....	257
3. Answers to Problems .....	263
Index .....	271