

Contents

Series foreword	xiii
Preface	xv
1 Basic set theory	1
1.1 Logical notation	1
1.2 Sets	2
1.2.1 Sets and properties	3
1.2.2 Some important sets	3
1.2.3 Constructions on sets	4
1.2.4 The axiom of foundation	6
1.3 Relations and functions	6
1.3.1 Lambda notation	7
1.3.2 Composing relations and functions	7
1.3.3 Direct and inverse image of a relation	9
1.3.4 Equivalence relations	9
1.4 Further reading	10
2 Introduction to operational semantics	11
2.1 IMP —a simple imperative language	11
2.2 The evaluation of arithmetic expressions	13
2.3 The evaluation of boolean expressions	17
2.4 The execution of commands	19
2.5 A simple proof	20
2.6 Alternative semantics	24
2.7 Further reading	26
3 Some principles of induction	27
3.1 Mathematical induction	27
3.2 Structural induction	28
3.3 Well-founded induction	31
3.4 Induction on derivations	35
3.5 Definitions by induction	39

3.6	Further reading	40
4	Inductive definitions	41
4.1	Rule induction	41
4.2	Special rule induction	44
4.3	Proof rules for operational semantics	45
4.3.1	Rule induction for arithmetic expressions	45
4.3.2	Rule induction for boolean expressions	46
4.3.3	Rule induction for commands	47
4.4	Operators and their least fixed points	52
4.5	Further reading	54
5	The denotational semantics of IMP	55
5.1	Motivation	55
5.2	Denotational semantics	56
5.3	Equivalence of the semantics	61
5.4	Complete partial orders and continuous functions	68
5.5	The Knaster-Tarski Theorem	74
5.6	Further reading	75
6	The axiomatic semantics of IMP	77
6.1	The idea	77
6.2	The assertion language Assn	80
6.2.1	Free and bound variables	81
6.2.2	Substitution	82
6.3	Semantics of assertions	84
6.4	Proof rules for partial correctness	89
6.5	Soundness	91
6.6	Using the Hoare rules—an example	93
6.7	Further reading	96
7	Completeness of the Hoare rules	99

7.1	Gödel's Incompleteness Theorem	99
7.2	Weakest preconditions and expressiveness	100
7.3	Proof of Gödel's Theorem	110
7.4	Verification conditions	112
7.5	Predicate transformers	115
7.6	Further reading	117
8	Introduction to domain theory	119
8.1	Basic definitions	119
8.2	Streams—an example	121
8.3	Constructions on cpo's	123
8.3.1	Discrete cpo's	124
8.3.2	Finite products	125
8.3.3	Function space	128
8.3.4	Lifting	131
8.3.5	Sums	133
8.4	A metalanguage	135
8.5	Further reading	139
9	Recursion equations	141
9.1	The language REC	141
9.2	Operational semantics of call-by-value	143
9.3	Denotational semantics of call-by-value	144
9.4	Equivalence of semantics for call-by-value	149
9.5	Operational semantics of call-by-name	153
9.6	Denotational semantics of call-by-name	154
9.7	Equivalence of semantics for call-by-name	157
9.8	Local declarations	161
9.9	Further reading	162
10	Techniques for recursion	163
10.1	Bekić's Theorem	163

10.2	Fixed-point induction	166
10.3	Well-founded induction	174
10.4	Well-founded recursion	176
10.5	An exercise	179
10.6	Further reading	181
11	Languages with higher types	183
11.1	An eager language	183
11.2	Eager operational semantics	186
11.3	Eager denotational semantics	188
11.4	Agreement of eager semantics	190
11.5	A lazy language	200
11.6	Lazy operational semantics	201
11.7	Lazy denotational semantics	203
11.8	Agreement of lazy semantics	204
11.9	Fixed-point operators	209
11.10	Observations and full abstraction	215
11.11	Sums	219
11.12	Further reading	221
12	Information systems	223
12.1	Recursive types	223
12.2	Information systems	225
12.3	Closed families and Scott predomains	228
12.4	A cpo of information systems	233
12.5	Constructions	236
12.5.1	Lifting	237
12.5.2	Sums	239
12.5.3	Product	241
12.5.4	Lifted function space	243
12.6	Further reading	249

13	Recursive types	251
13.1	An eager language	251
13.2	Eager operational semantics	255
13.3	Eager denotational semantics	257
13.4	Adequacy of eager semantics	262
13.5	The eager λ -calculus	267
	13.5.1 Equational theory	269
	13.5.2 A fixed-point operator	272
13.6	A lazy language	278
13.7	Lazy operational semantics	278
13.8	Lazy denotational semantics	281
13.9	Adequacy of lazy semantics	288
13.10	The lazy λ -calculus	290
	13.10.1 Equational theory	291
	13.10.2 A fixed-point operator	292
13.11	Further reading	295
14	Nondeterminism and parallelism	297
14.1	Introduction	297
14.2	Guarded commands	298
14.3	Communicating processes	303
14.4	Milner's CCS	308
14.5	Pure CCS	311
14.6	A specification language	316
14.7	The modal ν -calculus	321
14.8	Local model checking	327
14.9	Further reading	335
A	Incompleteness and undecidability	337
	Bibliography	353
	Index	357