

Contents

Chapter 1. Introduction	1
1.1. The 1-Categorical Story	2
1.2. Extended TFTs	4
1.3. Overview of Results	5
1.4. Notation	7
Acknowledgments	8
Chapter 2. Symplectic and Oriented Derived Stacks	9
2.1. Symplectic Derived Stacks	9
2.2. Oriented Derived Stacks	10
2.3. The ∞ -Categories of Presymplectic and Preoriented Stacks	12
2.4. Lagrangian Correspondences	16
2.5. Oriented Cospans	20
2.6. Higher Lagrangian Correspondences	23
2.7. Higher Oriented Cospans	28
2.8. Characterizations of Non-Degeneracy	31
2.9. Higher Categories of Symplectic and Oriented Derived Stacks	36
2.10. Symmetric Monoidal Structures	41
2.11. Full Dualizability	44
2.12. Oriented Cospans of Spaces	48
Chapter 3. The AKSZ Construction	53
3.1. Pushforward of Differential Forms	53
3.2. The AKSZ Construction on Differential Forms	55
3.3. The AKSZ Construction on Iterated Spans	60
3.4. Non-Degeneracy of the AKSZ Construction	63
Chapter 4. From Cobordisms to Cospans	65
4.1. Higher Categories of Bordisms	65
4.2. Cutting Bordisms: Idea	69
4.3. Cutting Bordisms: Details	71
4.4. Extended TFTs in Spans	76
Chapter 5. From Oriented Cobordisms to Oriented Cospans	79
5.1. From Cobordisms to Preoriented Spaces: Idea	79
5.2. Some Diagrams of Chain Complexes	82
5.3. A Model for Cospans in $\mathbf{Ch}_{\mathbb{K}/\mathbb{K}[-d]}$	88
5.4. The Integration Functor	92
5.5. Symmetric Monoidal Structure	98
5.6. Non-Degeneracy and TFTs in Lagrangian Correspondences	106

Appendix A. Background on Higher Categories	109
A.1. Iterated Segal Spaces	109
A.2. Higher Categories of Spans	111
A.3. Spans in Model Categories	116
A.4. Twisted Arrow ∞ -Categories and their Left Adjoint	118
Appendix B. Background on Derived Algebraic Geometry	123
B.1. Derived Rings	123
B.2. Cotangent Complexes of Algebras	125
B.3. Derived Stacks	127
B.4. Sheaves on Derived Stacks and \mathcal{O}_X -Modules	131
B.5. Quasicoherent Sheaves	134
B.6. Affine Morphisms and Relative Spec	138
B.7. Geometric Morphisms and Artin Stacks	141
B.8. Base Change and the Projection Formula	142
B.9. Naturality of Base Change and the Projection Formula	148
B.10. Cotangent Complexes of Derived Stacks	155
B.11. The de Rham Complex	162
B.12. Differential Forms	168
Bibliography	171