

Contents

Foreword	vii
Introduction	1
Acknowledgments	6
Chapter 1. Gagliardo–Nirenberg–Sobolev inequalities by variational methods	9
1.1. Gagliardo–Nirenberg–Sobolev inequalities	9
1.2. An elementary proof of the inequalities	11
1.3. Stability results by variational methods	24
1.4. Bibliographical comments	36
Chapter 2. The fast diffusion equation and the entropy methods	41
2.1. Gagliardo–Nirenberg–Sobolev inequalities and fast diffusion	41
2.2. Rényi entropy powers	43
2.3. The fast diffusion equation in self-similar variables	47
2.4. Refined entropy estimates	51
2.5. Bibliographical comments	56
Chapter 3. Linear parabolic equations: Interpolation and regularity	59
3.1. Interpolation inequalities and optimal constants	59
3.2. The constant in Moser’s Harnack inequality	66
3.3. Bibliographical comments	83
Chapter 4. Uniform convergence in relative error and threshold time	85
4.1. Statement	85
4.2. Local estimates and an interpolation estimate	86
4.3. Global Harnack Principle	99
4.4. Convergence in relative error and the threshold time t_*	102
4.5. Side results of interest and further observations	113
Chapter 5. Stability in Gagliardo–Nirenberg inequalities	117
5.1. Stability results in relative entropy	117
5.2. Stability of scale invariant Gagliardo–Nirenberg inequalities	119
5.3. Stability results: some remarks	122
Chapter 6. Constructive stability for Sobolev’s inequality	125
6.1. A result in the critical case	125
6.2. Time delay estimates	128
6.3. Entropy-entropy production inequalities and refinements	134
6.4. Uniform convergence in relative error	136
6.5. Computation of the stability constant	137

6.6. Concluding comments	138
Chapter 7. Discussion of the method	141
7.1. Decay properties	141
7.2. A limitation of the method	142
7.3. Boundedness of the second moment	143
7.4. Conclusion	146
Notations	147
Bibliography	153
Index	165