

Contents

Chapter 1. Introduction and main results	1
1.1. Historical context	1
1.2. The main theorems	3
1.3. An overview of the paper	8
1.4. Non-standard notation.	11
Chapter 2. General group theoretical results	13
2.1. Groups with a strongly p -embedded subgroup	14
Chapter 3. Maximal class p -groups	17
3.1. Automorphisms of p' -order	20
3.2. A theorem of Juhász	23
Chapter 4. Representations of groups with a cyclic Sylow p -subgroup	27
Chapter 5. A primer on fusion systems	33
Chapter 6. Fusion systems on p -groups of maximal class: Generalities	43
Chapter 7. Essential subgroups in exceptional maximal class p -groups	47
Chapter 8. The structure of $\gamma_1(S)$ when $\gamma_1(S)$ is \mathcal{F} -essential and S is not exceptional	53
Chapter 9. The structure of $\gamma_1(S)$ when $\gamma_1(S)$ is \mathcal{F} -essential and S is exceptional	57
Chapter 10. Locating \mathcal{F} -essential subgroups in groups of maximal class I	61
Chapter 11. Locating \mathcal{F} -essential subgroups in groups of maximal class II	67
11.1. The case T is S -centric	68
11.2. The case T is not S -centric	72
Chapter 12. The saturated fusion systems on exceptional maximal class groups: The proof of Theorem B	81
Chapter 13. The saturated fusion systems on non-exceptional maximal class groups	83
Chapter 14. The proofs of Theorems A and C	91
Chapter 15. A series of examples with non-abelian 2-step centralizer	95

Appendix A. Saturated fusion systems on maximal class p -groups with $\gamma_1(S)$ abelian	97
Appendix B. The saturated fusion systems on maximal class 3-groups	103
Appendix C. Computer code	107
A. The MAGMA code for Example 1.2	107
B. The MAGMA code required to verify Lemma 4.10	109
C. The MAGMA code for Lemma 7.5	110
D. The group providing an example for Lemma 8.5	111
E. The MAGMA code for Lemma B.3	112
Bibliography	113