

---

# Contents

Preface	ix
Chapter 1. Introductory models	1
§1.1. Saddle-node bifurcation in a budworm growth model	3
§1.2. Hopf bifurcation in the Rosenzweig-MacArthur model	6
§1.3. Homoclinic orbits and Sandstede's model	9
Chapter 2. Flows and invariant sets	15
§2.1. Invariant sets and attractors	17
§2.2. Periodic orbits	20
2.2.1. Floquet theory	20
2.2.2. Poincaré return maps	25
§2.3. Topological equivalence and bifurcation	30
§2.4. Singularity theory of smooth functions	36
§2.5. Smale horseshoe map	41
§2.6. Exercises	46
Chapter 3. Local bifurcations	51
§3.1. Methods of local bifurcation theory	52
3.1.1. Lyapunov-Schmidt reduction	53
3.1.2. Center manifold reduction	54
3.1.3. Normal forms	58
§3.2. Bifurcations of equilibria	59
3.2.1. Saddle-node bifurcation	60
3.2.2. Transcritical bifurcation and pitchfork bifurcation	69
3.2.3. Cusp bifurcation	85

3.2.4.	Hopf bifurcation	91
3.2.5.	Bogdanov-Takens bifurcation	107
§3.3.	Bifurcations of periodic orbits	135
3.3.1.	Saddle-node bifurcation of periodic orbits	135
3.3.2.	Period-doubling bifurcation	140
3.3.3.	Neimark-Sacker bifurcation	151
§3.4.	Exercises	169
Chapter 4.	Nonlocal bifurcations	177
§4.1.	Homoclinic orbits to equilibria	179
4.1.1.	Planar homoclinic orbits	180
4.1.2.	Higher-dimensional homoclinic orbits	195
4.1.3.	Saddle-focus homoclinic orbits	224
4.1.4.	Homoclinic orbit at a neutral saddle	230
§4.2.	Homoclinic orbits to periodic orbits	233
4.2.1.	Transverse homoclinic orbits	234
4.2.2.	Homoclinic tangencies	240
4.2.3.	Melnikov method	245
§4.3.	Exercises	250
Chapter 5.	Global bifurcations	257
§5.1.	Global structural stability	257
5.1.1.	Topological equivalence and bifurcation	258
5.1.2.	Hyperbolic sets	261
5.1.3.	Global stability theorems	270
5.1.4.	Robust structural instability	277
§5.2.	Bifurcation scenarios	282
5.2.1.	Surface flows	283
5.2.2.	Bifurcations in the Lorenz system	287
5.2.3.	Circle diffeomorphisms and torus flows	290
5.2.4.	Logistic and Hénon families	297
5.2.5.	Period-doubling cascades	313
5.2.6.	Intermittency	322
§5.3.	Exercises	331
Appendix A.	Elements of nonlinear analysis	335
§A.1.	Calculus on Banach spaces	335
A.1.1.	Derivatives	335
A.1.2.	Function spaces	337
A.1.3.	Power series expansions	340
A.1.4.	Superposition operators	342

---

§A.2. Uniform contractions and the implicit function theorem	345
A.2.1. Uniform contractions	346
A.2.2. Implicit function theorem	349
A.2.3. Lyapunov-Schmidt reduction	351
§A.3. Manifolds and transversality	353
A.3.1. Manifolds	353
A.3.2. Sard's theorem	356
Appendix B. Invariant manifolds and normal forms	357
§B.1. Invariant manifolds	357
B.1.1. Stable and unstable manifolds	357
B.1.2. Center manifolds	361
B.1.3. Strong stable manifolds and foliations	373
B.1.4. Center stable and center unstable manifolds	376
§B.2. Normal forms	378
B.2.1. The normal form theorem	378
B.2.2. Linearization	384
Appendix C. Lin's method	391
§C.1. Outline of Lin's method	392
§C.2. Exponential dichotomies and asymptotic expansions	397
C.2.1. Exponential dichotomies	398
C.2.2. Asymptotic expansions for orbits	410
§C.3. Proofs of Lin's method—1-periodic orbits	414
C.3.1. Existence of 1-periodic Lin orbits	415
C.3.2. Estimates of the jumps	428
C.3.3. Derivatives of the jumps	442
§C.4. Proofs of Lin's method—general case	450
C.4.1. Existence of Lin orbits	450
C.4.2. Estimates of the jumps	454
C.4.3. Derivatives of the jumps	461
§C.5. Forward and backward Lin orbits	473
§C.6. Bifurcation analysis with Lin's method	478
C.6.1. Existence of 1-periodic orbits	478
C.6.2. Stability analysis	481
C.6.3. Saddle-focus homoclinic orbits	488
Bibliography	495
Index	517