

# Contents

Chapter 1. Introduction	1
1.1. Unitary dual, conjugacy classes and coadjoint orbits	2
1.2. Nilpotent orbits and unitary representations	3
1.3. Theta Correspondence and Unipotent Representations	5
1.4. Main Results	6
Acknowledgments	9
Chapter 2. Invariants	11
2.1. Notation	11
2.2. Infinitesimal character and Harish-Chandra homomorphism	12
2.3. Leading exponents of irreducible representations	13
2.4. Global characters	16
2.5. Associated variety, asymptotic cycle, and wave front set	17
Chapter 3. Nilpotent orbits	19
3.1. Young diagrams and complex nilpotent orbits	19
3.2. Signed Young diagrams and real nilpotent orbits	21
3.3. Nilpotent orbits of class $\mathcal{U}$	23
3.4. Induced orbits	25
Chapter 4. Theta correspondences and quantum induction	33
4.1. Theta correspondence in semistable range	33
4.2. Unitarity and strongly semistable range	34
4.3. Quantum induction $\mathcal{Q}$	35
4.4. Unitary quantum induction $Q$	37
4.5. Moment map and $\Theta(G_1, G_2)$	39
4.6. Przebinda's results on $\mathcal{V}(\text{Ann } \theta(\pi))$	41
Chapter 5. A nonvanishing theorem	43
5.1. Nonvanishing of $\mathcal{Q}(2n_1; p, q; 2(p + q - n_1 - 1))(\pi)$ : Overview and main result	43
5.2. Results of Kudla-Rallis and Lee-Zhu	45
5.3. $Sp_{2n_1}(\mathbb{R}) \times Sp_{2n_2}(\mathbb{R})$ action on Lagrangian Grassmannian $X_{n_1+n_2}$	46
5.4. $I^\alpha(\pi)$ and $\text{Ind}_{MP_{n_2-n_1, n_1}}^{Mp_{2n_2}(\mathbb{R})} \pi^\tau \otimes \chi^\alpha$	50
5.5. Wave front set and the nonvanishing theorem	61
Chapter 6. Construction of unipotent representations, unitarity and infinitesimal character	65
6.1. Unitarity of $\mathcal{N}(\mathcal{O})$	66
6.2. Infinitesimal character	68

6.3.	$\mathcal{I}_{\pm}(\mathbf{d})$ and $\mathbf{d}$ : An algorithm	69
6.4.	Orderings and reversal phenomena	71
Chapter 7.	Associated varieties and existence of $\mathcal{N}(\mathcal{O}_{\mathbf{D}})$	73
7.1.	Estimates on infinitesimal characters: I	73
7.2.	Associated varieties of $\mathcal{N}(\mathcal{O}_{\mathbf{D}})$	74
7.3.	Nonvanishing of $\mathcal{N}(\mathcal{O}_{\mathbf{D}})$	75
Chapter 8.	Perspectives	77
8.1.	Wave front sets of $\mathcal{N}(\mathcal{O})$	77
8.2.	Wave front sets and induction functors	78
8.3.	Unitary dual, automorphic dual, and Ramanujan dual	79
Appendix		83
	Infinitesimal characters for $Mp$	83
	Integration of functions with values in Hilbert spaces	85
Bibliography		87