

# CONTENTS

<i>Symbols</i>	xi
<i>Introduction</i>	xii
<b>Part 1 Mathematics</b>	
<b>1 Calculation Procedures</b>	<b>3</b>
1.1 Calculation and advanced mathematics	3
1.2 Decisions about and within procedures	4
1.3 Learning from few (or no) examples	7
1.4 Generating your own exercises	10
1.5 Writing out calculations	11
1.6 Checking for errors	13
1.7 Mathematics is not just procedures	14
<b>2 Abstract Objects</b>	<b>19</b>
2.1 Numbers as abstract objects	19
2.2 Functions as abstract objects	21
2.3 What kind of object is that, really?	22
2.4 Objects as the results of procedures	24
2.5 Hierarchical organization of objects	26
2.6 Turning processes into objects	28
2.7 New objects: relations and binary operations	29
2.8 New objects: symmetries	31
<b>3 Definitions</b>	<b>36</b>
3.1 Axioms, definitions, and theorems	36
3.2 What are axioms?	37
3.3 What are definitions?	37
3.4 What are theorems?	39
3.5 Understanding definitions: even numbers	41
3.6 Understanding definitions: increasing functions	43

3.7	Understanding definitions: commutativity	46
3.8	Understanding definitions: open sets	48
3.9	Understanding definitions: limits	52
3.10	Definitions and intuition	52
<b>4</b>	<b>Theorems</b>	<b>59</b>
4.1	Theorems and logical necessity	59
4.2	A simple theorem about integers	61
4.3	A theorem about functions and derivatives	62
4.4	A theorem with less familiar objects	64
4.5	Logical language: “if”	67
4.6	Logical language: everyday uses of “if”	68
4.7	Logical language: quantifiers	70
4.8	Logical language: multiple quantifiers	72
4.9	Theorem rephrasing	74
4.10	Understanding: logical form and meaning	75
<b>5</b>	<b>Proof</b>	<b>79</b>
5.1	Proofs in high school mathematics	79
5.2	Proving that a definition is satisfied	80
5.3	Proving general statements	82
5.4	Proving general theorems using definitions	86
5.5	Definitions and other representations	88
5.6	Proofs, logical deductions, and objects	91
5.7	Proving obvious things	93
5.8	Believing counterintuitive things: the harmonic series	95
5.9	Believing counterintuitive things: Earth and rope	97
5.10	Will my whole major be proofs?	100
<b>6</b>	<b>Proof Types and Tricks</b>	<b>103</b>
6.1	General proving strategies	103
6.2	Direct proof	104
6.3	Proof by contradiction	108
6.4	Proof by induction	112
6.5	Uniqueness proofs	117
6.6	Adding and subtracting the same thing	119

6.7	Trying things out	121
6.8	“I would never have thought of that”	123
<b>7</b>	<b>Reading Mathematics</b>	<b>126</b>
7.1	Independent reading	126
7.2	Reading your lecture notes	128
7.3	Reading for understanding	129
7.4	Reading for synthesis	135
7.5	Using summaries for revision	139
7.6	Reading for memory	141
7.7	Using diagrams for memory	144
7.8	Reading proofs for memory	146
<b>8</b>	<b>Writing Mathematics</b>	<b>150</b>
8.1	Recognizing good writing	150
8.2	Why should a student write well?	153
8.3	Writing a clear argument	156
8.4	Using notation correctly	160
8.5	Arrows and brackets	163
8.6	Exceptions and mistakes	165
8.7	Separating out the task of writing	165
 <b>Part 2 Study Skills</b>		
<b>9</b>	<b>Lectures</b>	<b>171</b>
9.1	What are lectures like?	171
9.2	What are professors like?	172
9.3	Making lectures work for you	173
9.4	Tackling common problems	174
9.5	Learning in lectures	176
9.6	Courtesy in lectures	178
9.7	Feedback on lectures	181
<b>10</b>	<b>Other People</b>	<b>183</b>
10.1	Professors as teachers	183
10.2	Recitations and problems classes	184
10.3	Asking questions after and before lectures	185
10.4	Arranging a separate meeting with a professor	186

10.5	Asking questions electronically	188
10.6	(Mathematics) learning centers	189
10.7	Projects and internships	191
10.8	Studying with other students	193
10.9	Support with everything else	195
<b>11</b>	<b>Time Management</b>	<b>201</b>
11.1	Why would a good student read this chapter?	201
11.2	Aims and things to avoid	202
11.3	Planning for a semester	203
11.4	Planning for a typical week	208
11.5	Planning when to study what	215
11.6	Planning for an actual week	216
11.7	Where will you study?	219
11.8	Organizing your stuff	220
11.9	Not finishing things	220
<b>12</b>	<b>Panic</b>	<b>224</b>
12.1	Getting behind	224
12.2	What to do	225
<b>13</b>	<b>(Not) Being the Best</b>	<b>230</b>
13.1	Doing well as a mathematics major	230
13.2	What does understanding look like?	232
13.3	Keeping up	233
13.4	Understanding and speed	234
13.5	Not trying to understand everything	235
13.6	The mythical genius	237
<b>14</b>	<b>What Mathematics Professors Do</b>	<b>241</b>
14.1	When professors aren't lecturing	241
14.2	Teaching	242
14.3	Administration	242
14.4	Research	243
14.5	Becoming a mathematician	246
	<i>Bibliography</i>	248
	<i>Index</i>	263