

# Contents

|  |       |
|--|-------|
| <b>Preface</b> .....   | V     |
| <b>Highlights</b> .....  | IX    |
| <b>Application examples</b> .....  | XIX   |
| <b>Notation</b> .....  | XXIII |
| <b>1 Introduction to networked control systems</b> .....                   | 1     |
| 1.1 Networked systems .....  | 1     |
| 1.1.1 A control-theoretical view on cyber-physical systems .....           | 1     |
| 1.1.2 New fields of application .....                                      | 4     |
| 1.1.3 Characteristic properties of networked systems .....                 | 8     |
| 1.2 Networked control systems .....  | 9     |
| 1.2.1 Motivation for the control over digital networks .....               | 9     |
| 1.2.2 Properties of the communication network .....                        | 11    |
| 1.2.3 Flexibility of the information structure .....                       | 13    |
| 1.2.4 Challenges of networked systems design .....                         | 14    |
| 1.3 Survey over the control methods tackled in this book .....             | 17    |
| 1.3.1 Scope and key problems .....   | 17    |
| 1.3.2 Results of algebraic graph theory and network science .....          | 21    |
| 1.3.3 Consensus in networked systems .....                                 | 22    |
| 1.3.4 Synchronisation of multi-agent systems .....                         | 23    |
| 1.3.5 Design of the communication structure of networked controllers ..... | 24    |
| 1.3.6 Self-organisation in networked control systems .....                 | 25    |
| 1.3.7 Event-triggered control .....  | 26    |
| 1.3.8 Advanced control applications .....                                  | 26    |
| Notes and references .....   | 27    |
| <b>2 Algebraic graph theory</b> .....                                      | 31    |
| 2.1 Basic notions .....  | 31    |
| 2.1.1 Directed graphs .....  | 31    |
| 2.1.2 Other graphs .....   | 34    |
| 2.2 Matrix representations of graphs .....                                 | 36    |
| 2.2.1 Adjacency matrix .....   | 36    |
| 2.2.2 Degree matrix .....  | 38    |

|          |   |           |
|----------|---|-----------|
| 2.2.3    | Normalised adjacency matrix                                 | 39        |
| 2.2.4    | Incidence matrix  | 40        |
| 2.2.5    | Laplacian matrix  | 42        |
| 2.2.6    | Normalised Laplacian matrix                                 | 45        |
| 2.2.7    | Graph matrices in systems and control                       | 47        |
| 2.3      | Analysis of graphs  | 50        |
| 2.3.1    | Algebraic connectivity                                      | 50        |
| 2.3.2    | Reachability analysis                                       | 54        |
| 2.3.3    | Structure matrices  | 55        |
| 2.3.4    | Graphs associated with matrices                             | 56        |
| 2.3.5    | Specific graphs   | 57        |
| 2.4      | Summary   | 60        |
|          | Notes and references  | 61        |
| <b>3</b> | <b>Consensus in multi-agent systems</b>                     | <b>63</b> |
| 3.1      | Consensus problem   | 63        |
| 3.2      | Continuous-time consensus                                   | 67        |
| 3.2.1    | Basic convergence results                                   | 67        |
| 3.2.2    | Consensus value   | 71        |
| 3.2.3    | Convergence properties                                      | 75        |
| 3.3      | Specific multi-agent systems                                | 81        |
| 3.3.1    | Completely coupled systems                                  | 81        |
| 3.3.2    | Leader–follower networks                                    | 83        |
| 3.3.3    | Control-theoretical interpretation of the consensus problem | 85        |
| 3.3.4    | Compartmental systems                                       | 88        |
| 3.3.5    | Robust consensus systems                                    | 90        |
| 3.3.6    | Formation control   | 93        |
| 3.3.7    | Other versions of the consensus problem                     | 95        |
| 3.4      | Discrete-time consensus                                     | 98        |
| 3.4.1    | Consensus conditions  | 98        |
| 3.4.2    | Network of discrete-time integrators                        | 100       |
| 3.4.3    | Discrete-time version of continuous-time consensus systems  | 102       |
| 3.5      | Consensus over switching networks                           | 108       |
| 3.5.1    | Model   | 108       |
| 3.5.2    | Consensus over strongly connected networks                  | 109       |
| 3.5.3    | Consensus in systems with a changing number of agents       | 112       |
| 3.5.4    | Consensus over switching networks that are not connected    | 113       |
| 3.5.5    | Gossiping algorithms  | 118       |
| 3.6      | Summary   | 120       |
|          | Application studies   | 122       |
|          | Notes and references  | 123       |

|          |  |     |
|----------|--|-----|
| <b>4</b> | <b>Synchronisation of multi-agent systems</b>  | 127 |
| 4.1      | Synchronisation problem  | 127 |
| 4.2      | Models   | 130 |
| 4.2.1    | Agent model  | 130 |
| 4.2.2    | Networked controller   | 131 |
| 4.2.3    | Model of the networked system  | 132 |
| 4.3      | Asymptotic synchronisation of identical agents   | 133 |
| 4.3.1    | Output and state synchronisation   | 133 |
| 4.3.2    | Synchronisation condition  | 134 |
| 4.3.3    | Discussion of the synchronisation condition  | 140 |
| 4.3.4    | Synchronous trajectory   | 142 |
| 4.3.5    | Control-theoretical interpretation of the diffusive couplings                            | 145 |
| 4.3.6    | Design of synchronising controllers  | 151 |
| 4.3.7    | Synchronisation with complete couplings  | 153 |
| 4.4      | Synchronisable agents  | 160 |
| 4.4.1    | The notion of synchronisability  | 160 |
| 4.4.2    | Necessary condition for synchronisability  | 161 |
| 4.4.3    | Synchronising networks   | 162 |
| 4.4.4    | Synchronisation vs. consensus  | 164 |
| 4.5      | Example: Synchronising oscillator circuits   | 166 |
| 4.5.1    | Oscillator model   | 166 |
| 4.5.2    | Analysis of the circuit  | 167 |
| 4.5.3    | Diffusive couplings through resistor networks and synchronisation condition              | 169 |
| 4.5.4    | Behaviour of oscillator networks   | 170 |
| 4.6      | Synchronisation of agents with individual dynamics                                       | 174 |
| 4.6.1    | Model of the agents and of the networked system  | 174 |
| 4.6.2    | Synchronisability of individual agents   | 176 |
| 4.6.3    | Systems with common dynamics   | 177 |
| 4.6.4    | Synchronisability of non-identical agents  | 183 |
| 4.6.5    | Synchronisation condition  | 186 |
| 4.6.6    | Control-theoretical interpretation of the networked controller                           | 191 |
| 4.6.7    | Leader-follower networks   | 191 |
| 4.7      | Synchronisation of multi-agent systems by means of dynamical networked controllers       | 193 |
| 4.7.1    | Agents with local controllers  | 193 |
| 4.7.2    | Local controllers that make the agents synchronisable or improve the transient behaviour | 197 |
| 4.7.3    | Local controllers that introduce the virtual reference system into the agents            | 200 |
| 4.8      | Robustness of synchronised systems   | 206 |
| 4.8.1    | Three phenomena occurring in synchronised systems with parameter deviations              | 206 |
| 4.8.2    | Robustness for parameter deviations of the couplings                                     | 208 |

|          |  |            |
|----------|--|------------|
| 4.8.3    | Sensitivity of synchronised systems with respect to parameter deviations of the agents ..... | 208        |
| 4.8.4    | Practical synchronisation of agents with similar dynamics .....                              | 214        |
| 4.9      | Synchronisation of Kuramoto oscillators .....  | 220        |
| 4.9.1    | Oscillator models .....  | 220        |
| 4.9.2    | Asymptotic synchronisation of uniform Kuramoto oscillators .....                             | 223        |
| 4.9.3    | Phase-locking of non-uniform Kuramoto oscillators .....                                      | 225        |
| 4.9.4    | Asymptotic synchronisation of non-uniform oscillators by linear dynamical couplings .....    | 229        |
| 4.9.5    | Behaviour of Kuramoto oscillators with nonlinear couplings .....                             | 235        |
| 4.9.6    | Asymptotic synchronisation of non-uniform oscillators by nonlinear couplings .....           | 244        |
| 4.10     | Summary and outlook .....  | 253        |
|          | Application studies .....  | 255        |
|          | Notes and references .....   | 257        |
|          | Appendix 4.A: Proof of Lemma 4.6 .....   | 262        |
|          | Appendix 4.B: Proof of Lemma 4.7 .....   | 263        |
|          | Appendix 4.C: Proof of Lemma 4.8 .....   | 265        |
|          | Appendix 4.D: Proof of Lemma 4.10 .....  | 267        |
| <b>5</b> | <b>Design of the communication structure of networked controllers .....</b>                  | <b>269</b> |
| 5.1      | Control aims and design steps .....  | 269        |
| 5.1.1    | Efficient set-point following in multi-agent systems .....                                   | 269        |
| 5.1.2    | Main idea: Model abstraction .....   | 272        |
| 5.1.3    | Design steps .....   | 273        |
| 5.1.4    | Communication structure .....  | 274        |
| 5.2      | Models .....   | 278        |
| 5.2.1    | Agent model and reference system .....   | 278        |
| 5.2.2    | Local controllers .....  | 278        |
| 5.2.3    | Extended and controlled agents .....   | 279        |
| 5.2.4    | Overall system .....   | 281        |
| 5.2.5    | Main assumption and set-point following in cycle-free networks .....                         | 282        |
| 5.3      | Delay measures .....   | 283        |
| 5.3.1    | Motivation and definitions .....   | 283        |
| 5.3.2    | Determination of the delay .....   | 287        |
| 5.3.3    | Cumulative delay of networked systems .....  | 288        |
| 5.3.4    | Communication structure design .....   | 294        |
| 5.4      | Example: Distance control of vehicle platoons .....  | 296        |
| 5.4.1    | Control aims .....   | 296        |
| 5.4.2    | String stability of vehicle platoons .....   | 301        |
| 5.4.3    | Ideal vehicle dynamics for time-headway spacing .....  | 304        |
| 5.4.4    | Vehicle platoons with adaptive cruise controllers .....                                      | 306        |
| 5.4.5    | Vehicle platoons with cooperative adaptive cruise controllers .....                          | 309        |
| 5.4.6    | Vehicle dynamics for continuous progression and collision avoidance ..                       | 316        |
| 5.4.7    | Summary of the design objectives .....   | 323        |

|          |  |            |
|----------|--|------------|
| 5.4.8    | Specific vehicle controllers .....                                   | 325        |
| 5.5      | Summary .....  | 333        |
|          | Application studies .....  | 334        |
|          | Notes and references .....   | 335        |
| <b>6</b> | <b>Random graphs</b> .....   | <b>339</b> |
| 6.1      | Sparse graphs with random edges .....                                | 339        |
| 6.2      | Some further notions of graph theory .....                           | 341        |
| 6.2.1    | Characteristic path length .....                                     | 341        |
| 6.2.2    | Clustering coefficient .....   | 342        |
| 6.2.3    | Degree distribution .....  | 344        |
| 6.2.4    | Regular graphs .....   | 347        |
| 6.3      | Erdős-Rényi graphs .....   | 349        |
| 6.3.1    | Edge-generation rule for random graphs .....                         | 349        |
| 6.3.2    | Combinatorial properties of random graphs .....                      | 351        |
| 6.3.3    | Phase transitions in random graphs .....                             | 352        |
| 6.3.4    | Degree distribution and clustering coefficient .....                 | 356        |
| 6.3.5    | Diameter and characteristic path length .....                        | 358        |
| 6.4      | Results from Network Science .....                                   | 361        |
| 6.4.1    | Small-world architecture of complex networks .....                   | 361        |
| 6.4.2    | Survey over the models of complex networks .....                     | 362        |
| 6.4.3    | Small-world networks .....   | 364        |
| 6.4.4    | Scale-free networks .....  | 367        |
| 6.5      | Summary .....  | 371        |
|          | Application studies .....  | 374        |
|          | Notes and references .....   | 374        |
| <b>7</b> | <b>Consensus and synchronisation with random communication</b> ..... | <b>377</b> |
| 7.1      | Stochastic convergence .....   | 377        |
| 7.1.1    | Deterministic and random sequences .....                             | 377        |
| 7.1.2    | Convergence properties of random sequences .....                     | 380        |
| 7.2      | Stochastic stability of dynamical systems .....                      | 393        |
| 7.2.1    | Stochastic systems .....   | 393        |
| 7.2.2    | Martingales and supermartingales .....                               | 395        |
| 7.2.3    | Stability analysis .....   | 399        |
| 7.2.4    | Stability of fast switching systems .....                            | 405        |
| 7.3      | Random agreement .....   | 409        |
| 7.3.1    | General consensus condition .....                                    | 409        |
| 7.3.2    | Network of discrete-time integrators .....                           | 414        |
| 7.3.3    | Continuous consensus systems with random communication graph .....   | 415        |
| 7.3.4    | Random gossiping .....   | 418        |
| 7.4      | Random synchronisation .....   | 422        |
| 7.4.1    | Problem statement .....  | 422        |
| 7.4.2    | Synchronisation of fast switching systems .....                      | 423        |
| 7.4.3    | Synchronisation over a random graph .....                            | 426        |

|          |   |            |
|----------|---|------------|
| 7.5      | Summary .....   | 427        |
|          | Application studies .....   | 427        |
|          | Notes and references .....  | 428        |
|          | Appendix 7.A: Proof of Lemma 7.6 .....  | 429        |
| <b>8</b> | <b>Self-organisation in networked systems .....</b>   | <b>431</b> |
| 8.1      | The idea of self-organisation .....   | 431        |
| 8.2      | Self-connecting systems .....   | 434        |
|          | 8.2.1 Communication graphs with random links .....  | 434        |
|          | 8.2.2 Control aim and main assumptions .....  | 438        |
|          | 8.2.3 Properties of the effective communication graph .....   | 440        |
|          | 8.2.4 Communication with a restricted set of neighbours .....                                       | 450        |
| 8.3      | Small-world architecture of networked systems .....   | 457        |
|          | 8.3.1 Extending ring structures by shortcuts .....  | 457        |
|          | 8.3.2 Algorithm to generate the effective communication graph .....                                 | 459        |
|          | 8.3.3 Properties of the effective communication graph .....   | 462        |
|          | 8.3.4 Self-organised agents with changing leader .....  | 465        |
| 8.4      | Self-organised disturbance attenuation in multi-agent systems .....                                 | 467        |
|          | 8.4.1 Main idea: Cutting the communication links from disturbed towards<br>undisturbed agents ..... | 467        |
|          | 8.4.2 Communication structure .....   | 470        |
|          | 8.4.3 Controlled agents .....   | 471        |
|          | 8.4.4 Disturbance behaviour of the self-organised system .....                                      | 474        |
| 8.5      | Summary .....   | 480        |
|          | Application studies .....   | 481        |
|          | Notes and references .....  | 481        |
|          | Appendix 8.A: Proof of Lemma 4.7 .....  | 483        |
|          | Appendix 8.B: Proof of Lemma 8.6 .....  | 484        |
|          | Appendix 8.C: Proof of Theorem 8.2 .....  | 487        |
| <b>9</b> | <b>Event-triggered control .....</b>  | <b>489</b> |
| 9.1      | Motivation and control configuration .....  | 489        |
|          | 9.1.1 Structure of event-triggered control loops .....  | 489        |
|          | 9.1.2 Event-triggered control vs. sampled-data control .....  | 491        |
|          | 9.1.3 Main idea of event-triggered control .....  | 493        |
|          | 9.1.4 Continuous state-feedback control .....   | 495        |
| 9.2      | Event-triggered stabilisation .....   | 496        |
|          | 9.2.1 Stabilising state feedback .....  | 496        |
|          | 9.2.2 Event-triggered implementation of the controller .....  | 497        |
|          | 9.2.3 Properties of event-triggered control loops .....   | 502        |
|          | 9.2.4 Self-triggered implementation of the controller .....   | 505        |
|          | 9.2.5 Evaluation of the method .....  | 506        |
| 9.3      | Disturbance attenuation by event-triggered state feedback .....                                     | 508        |
|          | 9.3.1 Models .....  | 508        |
|          | 9.3.2 Behaviour of the control loop between two consecutive events .....                            | 510        |

|  |  |            |
|--|--|------------|
| 9.3.3  | Event generator and disturbance estimator .....  | 512        |
| 9.3.4  | Comparison of the event-triggered control loop and the continuous<br>state-feedback loop ..... | 517        |
| 9.3.5  | Communication frequency and minimum inter-event time .....                                     | 519        |
| 9.3.6  | Experimental results .....   | 520        |
| 9.4  | Event-triggered control of interconnected systems .....  | 526        |
| 9.4.1  | New structure and new phenomena .....  | 526        |
| 9.4.2  | Model of the interconnected system .....   | 528        |
| 9.4.3  | Decentralised event-triggered controllers .....  | 529        |
| 9.4.4  | Analysis of the event-triggered overall system .....   | 530        |
| 9.5  | Event-triggered synchronisation of multi-agent systems .....                                   | 536        |
| 9.5.1  | Structure of the overall system .....  | 536        |
| 9.5.2  | Components of event-triggered synchronising controllers .....                                  | 537        |
| 9.5.3  | Analysis of the overall system .....   | 540        |
| 9.6  | Summary and outlook .....  | 544        |
|  | Application studies .....  | 547        |
|  | Notes and references .....   | 547        |
| <b>References .....</b>                              |  | <b>551</b> |
| <b>Appendix 1: Solutions of the exercises .....</b>  |  | <b>569</b> |
| <b>Appendix 2: Matrices .....</b>                    |  | <b>669</b> |
| A2.1   | Series .....   | 669        |
| A2.2   | Inverse and pseudoinverse matrix .....   | 670        |
| A2.3   | Eigenvalues .....  | 670        |
| A2.4   | Specific matrices .....  | 672        |
| A2.5   | Stability analysis of complex-valued matrices .....  | 677        |
| A2.6   | Kronecker product .....  | 678        |
| <b>Appendix 3: Dynamical systems .....</b>           |  | <b>679</b> |
| A3.1   | Stability .....  | 679        |
| A3.2   | Further system properties .....  | 681        |
| A3.3   | Comparison principle .....   | 683        |
| <b>Appendix 4: Probability theory .....</b>          |  | <b>685</b> |
| A4.1   | Probability space .....  | 685        |
| A4.2   | Expectation and variance .....   | 688        |
| A4.3   | Random sequences .....   | 692        |
| A4.4   | Random matrices .....  | 694        |
| <b>Appendix 5: MATLAB functions for graphs .....</b> |  | <b>695</b> |
| A5.1   | Matrix functions .....   | 695        |
| A5.2   | Graph objects .....  | 697        |
| A5.3   | Graph algorithms .....   | 701        |

---

|  |     |
|--|-----|
| <b>Appendix 6: English–German dictionary</b> ..... | 709 |
| <b>Index</b> .....                                 | 713 |