Contents

Preface xi

Acknowledgments xv

1 Introduction 1

1.1 Previous constructions and Katz's theory of p-adic modular forms on the ordinary locus 1

1.2 Outline of our theory of p-adic analysis on the supersingular locus and construction of p-adic L-functions 6

1.3 Main results 23

1.4 Some remarks on other works in supersingular Iwasawa theory 25

2 Preliminaries: Generalities 27

2.1 Grothendieck sites and topoi 27

2.2 Pro-categories 29

2.3 Adic spaces 30

2.4 (Pre-)adic spaces and (pre)perfectoid spaces 39

2.5 Some complements on inverse limits of (pre-)adic spaces 43

2.6 The proétale site of an adic space 45

2.7 Period sheaves 49

2.8 The proétale "constant sheaf" $\hat{\mathbb{Z}}_{p,Y}$ 55

2.9 $\mathbb{B}^+_{\mathrm{dR},Y}$-local systems, $\mathcal{O}\mathbb{B}^+_{\mathrm{dR},Y}$-modules with connection, and the general de Rham comparison theorem 56

3 Preliminaries: Geometry of the infinite-level modular curve 63

3.1 The infinite-level modular curve 63

3.2 Relative étale cohomology and the Weil pairing 66

3.3 The $GL_2(\mathbb{Q}_p)$-action on $\mathcal{Y}$ (and $\hat{\mathcal{Y}}$) 67

3.4 The Hodge-Tate period and the Hodge-Tate period map 69

3.5 The Lubin-Tate period on the supersingular locus 72

3.6 The relative Hodge-Tate filtration 77

3.7 The fake Hasse invariant 78

3.8 Relative de Rham cohomology and the Hodge–de Rham filtration 79
3.9 Relative p-adic de Rham comparison theorem applied to $\mathcal{A} \to Y$ 80

4 The fundamental de Rham periods 83
4.1 A proétale local description of $\mathcal{O}\mathbb{B}_{\mathrm{dR}}^{(+)}$ 83
4.2 The fundamental de Rham periods 85
4.3 $GL_2(\mathbb{Q}_p)$-transformation properties of the fundamental de Rham periods 86
4.4 The p-adic Legendre relation 89
4.5 Relation to Colmez's "p-adic period pairing" 94
4.6 Relation to classical (Serre-Tate) theory on the ordinary locus 97
4.7 The Kodaira-Spencer isomorphism 109
4.8 The fundamental de Rham period $\mathbf{z}_{\mathrm{dR}}$ 112
4.9 The canonical differential 114

5 The p-adic Maass-Shimura operator 118
5.1 The "horizontal" lifting of the Hodge-Tate filtration 118
5.2 The "horizontal" relative Hodge-Tate decomposition over $\mathcal{O}_\Delta$ 122
5.3 Definition of the p-adic Maass-Shimura operator 126
5.4 The p-adic Maass-Shimura operator in coordinates and generalized p-adic modular forms 127
5.5 The p-adic Maass-Shimura operator with "nearly holomorphic coefficients" 131
5.6 The relative Hodge-Tate decomposition over $\mathcal{O}_\Delta^\dagger$ 138
5.7 The p-adic Maass-Shimura operator in coordinates and generalized p-adic nearly holomorphic modular forms 141
5.8 Relation of d_k^j and $(d_k^\dagger)^j$ to the ordinary Atkin-Serre operator $d_{k,\mathrm{AS}}^j$ and Katz's p-adic modular forms 147
5.9 Comparison between the complex and p-adic Maass-Shimura operators at CM points 150
5.10 Comparison of algeraic Maass-Shimura derivatives on different levels 159

6 p-adic analysis of the p-adic Maass-Shimura operators 162
6.1 q_{dR}-expansions 162
6.2 Relation between q_{dR}-expansions and Serre-Tate expansions 172
6.3 Integrality properties of q_{dR}-expansions: the Dieudonné-Dwork lemma 174
6.4 Integral structures on stalks of intermediate period sheaves between $\mathcal{O}_\Delta$ and $\hat{\mathcal{O}}_\Delta$ 180

6.5 The p-adic Maass-Shimura operator θ_k^j in q_{dR}-coordinates 182
6.6 Integrality of q_{dR}-expansions and the $\flat$-operator 184
6.7 p-adic analytic properties of p-adic Maass-Shimura operators 186

7 Bounding periods at supersingular CM points 197
7.1 Periods of supersingular CM points 197
7.2 Weights 205
7.3 Good CM points 208

8 Supersingular Rankin-Selberg p-adic L-functions 216
8.1 Preliminaries for the construction 216
8.2 Construction of the p-adic L-function 219
8.3 Interpolation 228
8.3.1 Interpolation formula 228
8.3.2 Interpolation formula 234

9 The p-adic Waldspurger formula 236
9.1 Coleman integration 237
9.2 Coleman primitives in our situation 237
9.3 The p-adic Waldspurger formula 243
9.4 p-adic Kronecker limit formula 248

Bibliography 251

Index 257