

# Contents

<b>Preface</b>	<b>ix</b>
Aims and Contents . . . . .	ix
Mathematical and Scholarly Level . . . . .	xi
Acknowledgments . . . . .	xii
<b>1 Nim and Combinatorial Games</b>	<b>1</b>
1.1 Prerequisites and Learning Outcomes . . . . .	1
1.2 Nim . . . . .	2
1.3 Top-down Induction . . . . .	7
1.4 Game Sums and Equivalence of Games . . . . .	10
1.5 Nim, Poker Nim, and the Mex Rule . . . . .	15
1.6 Sums of Nim Heaps . . . . .	19
1.7 Finding Nim Values . . . . .	22
1.8 A Glimpse of Partizan Games . . . . .	24
1.9 Further Reading . . . . .	30
1.10 Exercises for Chapter 1 . . . . .	32
<b>2 Congestion Games</b>	<b>39</b>
2.1 Prerequisites and Learning Outcomes . . . . .	40
2.2 Introduction: The Pigou Network . . . . .	40
2.3 The Braess Paradox . . . . .	43
2.4 Definition of Congestion Games . . . . .	44
2.5 Existence of Equilibrium in a Congestion Game . . . . .	46
2.6 Atomic and Splittable Flow, Price of Anarchy . . . . .	48
2.7 Further Reading . . . . .	49
2.8 Exercises for Chapter 2 . . . . .	50
<b>3 Games in Strategic Form</b>	<b>53</b>
3.1 Prerequisites and Learning Outcomes . . . . .	54
3.2 Games in Strategic Form . . . . .	54
3.3 Best Responses and Equilibrium . . . . .	57
3.4 Games with Multiple Equilibria . . . . .	60
3.5 Dominated Strategies . . . . .	62

3.6	The Cournot Duopoly of Quantity Competition . . . . .	66
3.7	Games without a Pure-Strategy Equilibrium . . . . .	72
3.8	Symmetric Games with Two Strategies per Player . . . . .	72
3.9	Further Reading . . . . .	74
3.10	Exercises for Chapter 3 . . . . .	75
<b>4</b>	<b>Game Trees with Perfect Information</b>	<b>77</b>
4.1	Prerequisites and Learning Outcomes . . . . .	78
4.2	Definition of Game Trees . . . . .	79
4.3	Backward Induction . . . . .	82
4.4	Strategies in Game Trees . . . . .	84
4.5	Reduced Strategies . . . . .	86
4.6	Subgame-Perfect Equilibrium (SPE) . . . . .	91
4.7	Commitment Games . . . . .	93
4.8	Further Reading . . . . .	98
4.9	Exercises for Chapter 4 . . . . .	99
<b>5</b>	<b>Expected Utility</b>	<b>103</b>
5.1	Prerequisites and Learning Outcomes . . . . .	105
5.2	Summary . . . . .	106
5.3	Decisions Under Risk . . . . .	109
5.4	Preferences for Lotteries . . . . .	111
5.5	Ordinal Preferences for Decisions Under Certainty . . . . .	113
5.6	Cardinal Utility Functions and Simple Lotteries . . . . .	115
5.7	Consistency Axioms . . . . .	120
5.8	Existence of an Expected-Utility Function . . . . .	124
5.9	Risk Aversion . . . . .	129
5.10	Discussion and Further Reading . . . . .	132
5.11	Exercises for Chapter 5 . . . . .	134
<b>6</b>	<b>Mixed Equilibrium</b>	<b>135</b>
6.1	Prerequisites and Learning Objectives . . . . .	136
6.2	Compliance Inspections . . . . .	136
6.3	Bimatrix Games . . . . .	141
6.4	The Best-Response Condition . . . . .	145
6.5	Existence of Mixed Equilibria . . . . .	149
6.6	Finding Mixed Equilibria in Small Games . . . . .	153
6.7	The Upper-Envelope Method . . . . .	158
6.8	Degenerate Games . . . . .	161
6.9	Further Reading . . . . .	166
6.10	Exercises for Chapter 6 . . . . .	167

<b>7</b>	<b>Brouwer's Fixed-Point Theorem</b>	<b>169</b>
7.1	Prerequisites and Learning Outcomes . . . . .	171
7.2	Labels . . . . .	171
7.3	Simplices and Triangulations . . . . .	173
7.4	Sperner's Lemma . . . . .	180
7.5	The Knaster–Kuratowski–Mazurkiewicz Lemma . . . . .	187
7.6	Brouwer's Fixed-Point Theorem on a General Compact Convex Set	190
7.7	The Freudenthal Triangulation . . . . .	196
7.8	Further Reading . . . . .	203
7.9	Exercises for Chapter 7 . . . . .	204
<b>8</b>	<b>Zero-Sum Games</b>	<b>205</b>
8.1	Prerequisites and Learning Outcomes . . . . .	207
8.2	Example: Soccer Penalty . . . . .	207
8.3	Max-Min and Min-Max Strategies . . . . .	211
8.4	A Short Proof of the Minimax Theorem . . . . .	216
8.5	Further Notes on Zero-Sum Games . . . . .	218
8.6	Further Reading . . . . .	220
8.7	Exercises for Chapter 8 . . . . .	221
<b>9</b>	<b>Geometry of Equilibria in Bimatrix Games</b>	<b>223</b>
9.1	Prerequisites and Learning Outcomes . . . . .	224
9.2	Labeled Best-Response Regions . . . . .	224
9.3	The Lemke–Howson Algorithm . . . . .	230
9.4	Using Best-Response Diagrams . . . . .	235
9.5	Strategic Equivalence . . . . .	239
9.6	Best-Response Polyhedra and Polytopes . . . . .	245
9.7	Complementary Pivoting . . . . .	250
9.8	Degeneracy Resolution . . . . .	256
9.9	Further Reading . . . . .	259
9.10	Exercises for Chapter 9 . . . . .	261
<b>10</b>	<b>Game Trees with Imperfect Information</b>	<b>263</b>
10.1	Prerequisites and Learning Outcomes . . . . .	264
10.2	Information Sets . . . . .	264
10.3	Extensive Games . . . . .	270
10.4	Strategies for Extensive Games and the Strategic Form . . . . .	272
10.5	Reduced Strategies . . . . .	274
10.6	Perfect Recall . . . . .	275
10.7	Behavior Strategies . . . . .	279
10.8	Kuhn's Theorem: Behavior Strategies Suffice . . . . .	283

10.9	Behavior Strategies in the Monty Hall Problem . . . . .	287
10.10	Subgames and Subgame-Perfect Equilibria . . . . .	291
10.11	Further Reading . . . . .	295
10.12	Exercises for Chapter 10 . . . . .	296
<b>11</b>	<b>Bargaining</b> . . . . .	<b>299</b>
11.1	Prerequisites and Learning Outcomes . . . . .	300
11.2	Bargaining Sets . . . . .	300
11.3	Bargaining Axioms . . . . .	302
11.4	The Nash Bargaining Solution . . . . .	305
11.5	Geometry of the Bargaining Solution . . . . .	308
11.6	Splitting a Unit Pie . . . . .	311
11.7	The Ultimatum Game . . . . .	313
11.8	Alternating Offers Over Two Rounds . . . . .	317
11.9	Alternating Offers Over Several Rounds . . . . .	320
11.10	Stationary Strategies . . . . .	324
11.11	The Nash Bargaining Solution Via Alternating Offers . . . . .	328
11.12	Further Reading . . . . .	330
11.13	Exercises for Chapter 11 . . . . .	330
<b>12</b>	<b>Correlated Equilibrium</b> . . . . .	<b>333</b>
12.1	Prerequisites and Learning Outcomes . . . . .	333
12.2	Examples of Correlated Equilibria . . . . .	334
12.3	Incentive Constraints . . . . .	337
12.4	Coarse Correlated Equilibrium . . . . .	341
12.5	Existence of a Correlated Equilibrium . . . . .	344
12.6	Further Reading . . . . .	348
12.7	Exercises for Chapter 12 . . . . .	349
	<b>References</b> . . . . .	<b>351</b>
	<b>Index</b> . . . . .	<b>358</b>