

Contents

1	Geometry of Many-Body Systems	1
1.1	Planar Many-Body Systems.....	1
1.2	Rotation and Vibration of Planar Many-Body Systems.....	10
1.3	Vibrations Induce Rotations in Two Dimensions	12
1.4	Planar Three-Body Systems.....	16
1.5	The Rotation Group $SO(3)$	23
1.6	Spatial Many-Body Systems	33
1.7	Rotation and Vibration for Spatial Many-Body Systems	42
1.8	Local Description of Spatial Many-Body Systems	51
1.8.1	Local Product Structure.....	51
1.8.2	Local Description in the Space Frame	53
1.8.3	Local Description in the Rotated Frame	57
1.9	Spatial Three-Body Systems	60
1.10	Non-separability of Vibration from Rotation.....	65
2	Mechanics of Many-Body Systems	69
2.1	Equations of Motion for a Free Rigid Body	69
2.2	Variational Principle for a Free Rigid Body	74
2.3	Lagrangian Mechanics of Many-Body Systems	78
2.4	Hamel's Approach	86
2.5	Hamiltonian Mechanics of Many-Body Systems	89
3	Mechanical Control Systems	95
3.1	Electron Motion in an Electromagnetic Field	95
3.2	The Inverted Pendulum on a Cart.....	100
3.3	Port-Hamiltonian Systems	105
3.4	Remarks on Optimal Hamiltonians	108
4	The Falling Cat	111
4.1	Modeling of the Falling Cat	111
4.2	Geometric Setting for Rigid Body Systems	112

4.3	Geometric Setting for Two Jointed Cylinders	114
4.3.1	The Configuration Space	114
4.3.2	Geometric Quantities	117
4.3.3	Summary and a Remark on the Geometric Setting	122
4.4	A Lagrangian Model of the Falling Cat	124
4.5	A Port-Controlled Hamiltonian System	126
4.6	Execution of Somersaults	129
4.7	Remarks on Control Problems	133
5	Appendices	137
5.1	Newton's Law of Gravitation, Revisited	137
5.2	Principal Fiber Bundles	142
5.3	Spatial N -Body Systems with $N \geq 4$	143
5.4	The Orthogonal Group $O(n)$	152
5.5	Many-Body Systems in n Dimensions	155
5.6	Holonomy for Many-Body Systems	162
5.7	Rigid Bodies in n Dimensions	164
5.8	Kaluza–Klein Formalism	169
5.9	Symplectic Approach to Hamilton's Equations	172
5.10	Remarks on Related Topics	174
5.10.1	Quantum Many-Body Systems	174
5.10.2	Geometric Phases and Further Reading	175
5.10.3	Open Dynamical Systems and Developments	176
	Bibliography	177
	Index	181