

Contents

1	Some Elements on Berthelot's Arithmetic $\mathcal{D}$-Modules	1
	Daniel Caro	
1.1	Introduction	1
	Introduction	1
1.2	Ring of Differential Operators	2
1.2.1	Kähler Differential	2
1.2.2	Ring of Differential Operators	4
1.2.3	Smooth Differential Case	6
1.2.4	Left $D_{A/R}$ -Modules	8
1.3	Berthelot's Ring of Differential Operators of Finite Level and Infinite Order	12
1.3.1	Formal Affine Case, Standard Ring of Differential Operators	12
1.3.2	Formal Affine Case, Berthelot's Ring of Differential Operators of Level m	14
1.3.3	Left $D_{A/R}^{(m)}$ -modules	19
1.3.4	Weak p -Adic Completion	23
1.3.5	Sheafification, Coherence	25
	Appendix: Further Reading	27
	References	28
2	Difference Galois Theory for the "Applied" Mathematician	29
	Lucia Di Vizio	
2.1	Introduction	29
2.2	Glossary of Difference Algebra	31
2.3	Picard-Vessiot Rings	33
2.4	The Galois Group	37
2.4.1	A Short Digression on Group Schemes	38
2.4.2	The Galois Group of a Linear Difference System	40
2.4.3	Transcendence Degree of the Picard-Vessiot Extension	42
2.5	The Galois Correspondence (First Part)	44

2.6	Application to Transcendence and Differential Transcendence	45
2.6.1	General Statements	45
2.6.2	Differential Algebraicity and D -Finiteness	47
2.7	Applications to Special Cases	48
2.7.1	Finite Difference Equations and Hölder Theorem	49
2.7.2	Linear Inhomogeneous q -Difference Equations of the First Order	51
2.7.3	A Particular Case of the Ishizaki-Ogawara's Theorem	53
2.8	The Galois Correspondence (Second Part)	54
	Appendix: Behavior of the Galois Group with Respect to the Iteration of τ	57
	References	58
3	Igusa's Conjecture on Exponential Sums Modulo p^m and the Local-Global Principle	61
	Kien Huu Nguyen	
3.1	Introduction	61
3.2	Igusa Local Zeta Functions and Exponential Sums Modulo p^m	68
3.2.1	Local Fields	68
3.2.2	Embedded Resolutions	70
3.2.3	Igusa Local Zeta Functions and the Monodromy Conjecture	71
3.2.4	Exponential Sums and Fiber Integration	74
3.3	Igusa's Conjecture on Exponential Sums Modulo p^m	76
3.3.1	Adèles	76
3.3.2	Poisson Formulas and Formulas of Siegel Type	78
3.3.3	Some Expected Results	81
3.4	Progress on Igusa's Conjecture	82
3.4.1	The Non-degenerate Case	82
3.4.2	Beyond the Non-degenerate Case	84
3.5	A Long History of the Local-Global Principle	85
3.5.1	The Local-Global Principle	85
3.5.2	Progress on the Local-Global Principle	86
3.5.3	Igusa's Approach	88
	References	90
4	From the Carlitz Exponential to Drinfeld Modular Forms	93
	Federico Pellarin	
4.1	Introduction	93
4.2	Rings and Fields	95
4.2.1	Local Compactness, Local Fields	96
4.2.2	Valued Rings and Fields for Modular Forms	98
4.2.3	Algebraic Extensions	100
4.2.4	Analytic Functions on Disks	102
4.2.5	Further Properties of the Field $\mathbb{C}_\infty$	103
4.3	Drinfeld Modules and Uniformisation	106

4.3.1	Drinfeld A -Modules and A -Lattices	106
4.3.2	From Drinfeld Modules to Exponential Functions	109
4.4	The Carlitz Module and Its Exponential	113
4.4.1	A Formula for v	115
4.4.2	A Factorization Property for the Carlitz Exponential	119
4.4.3	The Function $\exp_A$ and Local Class Field Theory	123
4.5	Topology of the Drinfeld Upper-Half Plane	127
4.5.1	Rigid Analytic Spaces	129
4.5.2	Fundamental Domains for $\Gamma \backslash \Omega$	133
4.5.3	An Elementary Result on Translation-Invariant Functions Over Ω	140
4.6	Some Quotient Spaces	142
4.6.1	A -Periodic Functions Over Ω	142
4.6.2	The Quotient $GL_2(A) \backslash \Omega$	149
4.7	Drinfeld Modular Forms	151
4.7.1	u -Expansions	152
4.7.2	Construction of Non-trivial Cusp Forms	156
4.8	Eisenstein Series with Values in Banach Algebras	159
4.9	Modular Forms with Values in Banach Algebras	165
4.9.1	Weak Modular Forms of Weight -1	167
4.9.2	Jacobi-Like Forms	172
	References	175
5	Berkovich Curves and Schottky Uniformization I: The Berkovich Affine Line	179
	Jérôme Poineau and Daniele Turchetti	
5.1	Introduction	179
5.2	The Underlying Set	181
5.3	Classification of Points	184
5.4	Topology	190
5.5	Analytic Functions	197
5.6	Extension of Scalars	201
5.7	Connectedness	206
5.8	Virtual Discs and Annuli	211
5.8.1	Definitions	211
5.8.2	The Case of an Algebraically Closed and Maximally Complete Base Field	213
5.8.3	The General Case	214
5.9	Lengths of Intervals	215
5.10	Variation of Rational Functions	217
	References	223
6	Berkovich Curves and Schottky Uniformization II: Analytic Uniformization of Mumford Curves	225
	Jérôme Poineau and Daniele Turchetti	
6.1	Introduction	225

6.2	The Berkovich Projective Line and Möbius Transformations	227
6.2.1	Affine Berkovich Spaces	227
6.2.2	The Berkovich Projective Line	228
6.2.3	Möbius Transformations	229
6.2.4	Loxodromic Transformations and Koebe Coordinates	231
6.3	Berkovich k -Analytic Curves	232
6.3.1	Berkovich $\mathbb{A}^1$ -like Curves	232
6.3.2	Arbitrary Smooth Curves	235
6.3.3	Mumford Curves	239
6.4	Schottky Groups	241
6.4.1	Schottky Figures	241
6.4.2	Group-Theoretic Version	250
6.4.3	Twisted Ford Discs	252
6.4.4	Local Fields	255
6.5	Uniformization of Mumford Curves	256
6.5.1	The Uniformization Theorem	257
6.5.2	Automorphisms of Mumford Curves	264
	Appendix: Further Reading	270
A.1	Berkovich Spaces and their Skeleta	271
A.2	Non-Archimedean Uniformization in Arithmetic Geometry	272
A.3	The Relevance of Mumford Curves	274
	References	277
7	On the Stark Units of Drinfeld Modules	281
	Florin Tavares Ribeiro	
7.1	Introduction	281
7.2	Background	283
7.2.1	Notation	283
7.2.2	Fitting Ideals	284
7.2.3	Ratio of Covolumes	286
7.3	Drinfeld Modules	287
7.3.1	Drinfeld Modules	288
7.3.2	Exponential and Logarithm	288
7.3.3	The Carlitz Module	290
7.4	Stark Units	291
7.4.1	Taelman Modules	291
7.4.2	The Module of Stark Units	293
7.4.3	Link with Anderson's Special Points	296
7.5	Class Formulas	297
7.5.1	Taelman's Class Formula	297
7.5.2	The Equivariant Class Formula	300
7.5.3	Examples	302
7.6	The Multi-Variable Deformation of a Drinfeld A -Module	304
7.6.1	The Multi-Variable Setting	304
7.6.2	The Canonical Deformation of the Carlitz Module	307

7.7	Applications.....	311
7.7.1	Discrete Greenberg Conjectures	311
7.7.2	On the Bernoulli-Carlitz Numbers.....	317
7.8	Stark Units in More General Settings.....	321
	References	323