

Mathematics for Enzyme Reaction Kinetics and Reactor Performance

Volume 1

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You cannot teach a man anything; you can only help him find it for himself.
Galileo Galilei

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