

# Contents

Chapter 1. Introduction	1
Chapter 2. Some abstract 2-algebra	9
2.1. Rectangular 2-algebras	9
2.2. Rectangular algebra-modules and 2-modules	12
2.3. Motility hypotheses and tensor products	15
2.4. Sequential objects and restricted tensor products	20
2.5. Module-2-modules, algebra-bimodules, and bimodule-modules	24
Chapter 3. More 2-algebra: Bending and smoothing	27
3.1. The top-right bent tensor product	27
3.2. 2-modules as bent modules	30
3.3. The smoothed tensor product	33
3.4. The bottom-left bent tensor product	33
3.5. $A_\infty$ -2-modules	34
Chapter 4. Some homological algebra of 2-modules	35
Chapter 5. The algebras and algebra-modules	39
5.1. The algebra associated to a matched circle	39
5.2. The algebra-modules associated to matched intervals	41
5.3. Gluing surfaces with boundary	44
Chapter 6. The cornering module-2-modules	47
6.1. The $DD$ identity module-bimodule	47
6.2. The $D$ - $AA$ - and $A$ - $DD$ -cornering modules	52
6.3. The other cornering modules	55
Chapter 7. The trimodules $\mathbb{T}_{DDD}$ and $\mathbb{T}_{DDA}$	57
7.1. Combinatorial descriptions of the trimodules	59
7.2. Computation of $\mathbb{T}_{DDD}$	63
7.3. Computation of $\mathbb{T}_{DDA}$	69
Chapter 8. Cornered 2-modules for cornered Heegaard diagrams	71
8.1. Cornered Heegaard diagrams	71
8.2. Definition of the cornered 2-modules	72
8.3. Tensor products of cornering module-2-modules	73
8.4. Proofs of the invariance and gluing theorems	76
Chapter 9. Gradings	79
9.1. Noncommutative gradings	79
9.2. Gradings on the cornered 2-algebras, algebra-modules, and 2-modules	81

9.3. The graded pairing theorem	88
Chapter 10. Practical computations	93
10.1. Induction and restriction functors	93
10.2. The multiplicity-one 2-algebra	95
10.3. An example	97
Chapter 11. The nilCoxeter planar algebra	109
Bibliography	111