

Model Theory and the Philosophy of Mathematical Practice

Formalization without Foundationalism

JOHN T. BALDWIN

University of Illinois, Chicago



Contents

List of Figures [page x]

Acknowledgments [xi]

Introduction [1]

PART I REFINING THE NOTION OF CATEGORICITY [29]

- 1 Formalization [31]
 - 1.1 The Concept of Formalization [32]
 - 1.2 Vocabulary and Structures [34]
 - 1.3 Logics [41]
 - 1.4 Theories and Axioms [47]
- 2 The Context of Formalization [51]
 - 2.1 The Process of Formalization [51]
 - 2.2 Two Roles of Formalization [54]
 - 2.3 A Criterion for Evaluating Properties of Theories [58]
 - 2.4 Virtuous Properties as an Organizing Principle [61]
- 3 Categoricity [68]
 - 3.1 Categoricity of Second Order Theories [69]
 - 3.2 $L_{\omega_1, \omega}$ -categoricity [74]
 - 3.3 $L_{\omega, \omega}$: Categoricity in Power [78]
 - 3.4 The Significance of Categoricity (in Power) [84]

PART II THE PARADIGM SHIFT [87]

- 4 What Was Model Theory About? [89]
 - 4.1 The Downward Löwenheim–Skolem–Tarski Theorem [89]
 - 4.2 Completeness, Compactness, and the Upward Löwenheim–Skolem–Tarski Theorem [92]
 - 4.3 Complete Theories [99]
 - 4.4 Quantifier Complexity [104]
 - 4.5 Interpretability [108]
 - 4.6 What Is a Structure, Really? [111]
 - 4.7 When Are Structures ‘Equal’? [116]

- 5 What Is Contemporary Model Theory About? [119]
 - 5.1 Analogy to Theorem to Method [119]
 - 5.2 Universal Domains [124]
 - 5.3 The Stability Hierarchy [128]
 - 5.4 Combinatorial Geometry [133]
 - 5.5 Classification: The Main Gap [137]
 - 5.6 Why Is Model Theory So Entwined with Classical Mathematics? [143]
- 6 Isolating Tame Mathematics [148]
 - 6.1 Groups of Finite Morley Rank [149]
 - 6.2 Formal Methods as a Tool in Mathematics [151]
 - 6.3 First Order Analysis [156]
 - 6.4 What Are the Central Notions of Model Theory? [162]
- 7 Infinitary Logic [167]
 - 7.1 Categoricity in Uncountable Power for $L_{\omega_1, \omega}$ [168]
 - 7.2 The Vaught Conjecture [171]
 - 7.3 Déjà vu: Categoricity in Infinitary Second Order Logic [175]
- 8 Model Theory and Set Theory [177]
 - 8.1 Is There Model Theory without Axiomatic Set Theory? [178]
 - 8.2 Is There Model Theory without Combinatorial Set Theory? [182]
 - 8.3 Why Is \aleph_0 Exceptional for Model Theory? [186]
 - 8.4 Entanglement of Model Theory and Cardinality [189]
 - 8.5 Entanglement of Model Theory and the Replacement Axiom [192]
 - 8.6 Entanglement of Model Theory with Extensions of ZFC [196]
 - 8.7 Moral [198]

PART III GEOMETRY [201]

- 9 Axiomatization of Geometry [203]
 - 9.1 The Goals of Axiomatization [205]
 - 9.2 Descriptions of the Geometric Continuum [211]
 - 9.3 Some Geometric Data Sets and Axiom Systems [217]
 - 9.4 Geometry and Algebra [221]
 - 9.5 Proportion and Area [229]
- 10 π , Area, and Circumference of Circles [234]
 - 10.1 π in Euclidean and Archimedean Geometry [234]
 - 10.2 From Descartes to Tarski [239]
 - 10.3 π in Geometries over Real Closed Fields [243]
- 11 Complete: The Word for All Seasons [250]
 - 11.1 Hilbert's Continuity Axioms [252]
 - 11.2 Against the Dedekind Postulate for Geometry [255]

PART IV	METHODOLOGY	[259]
12	Formalization and Purity in Geometry	[261]
12.1	Content and Vocabulary	[262]
12.2	Projective and Affine Geometry	[265]
12.3	General Schemes for Characterizing Purity	[267]
12.4	Modesty, Purity, and Generalization	[273]
12.5	Purity and the Desargues Proposition	[273]
12.6	Distinguishing Algebraic and Geometric Proof	[281]
13	On the Nature of Definition: Model Theory	[283]
13.1	Methodology of Classification	[285]
13.2	The Fecundity of the Stability Hierarchy	[287]
13.3	Dividing Lines	[292]
13.4	Definition, Classification, and Taxonomy	[294]
14	Formalism-Freeness (Mathematical Properties)	[300]
15	Summation	[312]
	<i>References</i>	[317]
	<i>Index</i>	[347]