

# Contents

---

## 1 First-Order Equations

---

1.1	Introduction	1
1.2	Linear Equations	7
1.3	Nonhomogeneous Linear Equations	14
1.4	Further Applications of Linear Equations	22
1.5	Separable Equations	32
1.6	Equations Reducible to Separable	40
1.7	Graphical Methods	49
1.7A	Theoretical Matters	57
1.8	Exact Equations	62
1.9	Integrating Factors	70
	Notes and References	75
	Miscellaneous Exercises	75

## 2 Second-Order Linear Equations: Basic Methods and Applications

---

2.1	Introduction	79
2.2	Solution of Homogeneous Equations with Constant Coefficients	86
2.3	Free Vibrations	98
2.4	Nonhomogeneous Equations—Undetermined Coefficients	110
2.5	Forced Vibrations	119
2.6	Electrical Circuits	129
	Notes	135
	Miscellaneous Exercises	135

## 3 Theory of Linear Equations

---

3.1	Theory of Linear Equations	138
3.2	The Solution of Equations with Constant Coefficients	149
3.3	The Euler-Cauchy Equation	157

3.4	Reduction of Order	163
3.5	Variation of Parameters	168
3.6	Second-Order Linear Equations with Variable Coefficients	177
	Notes and References	186
	Miscellaneous Exercises	186

## 4 Power Series Methods

---

4.1	Power Series Solutions	189
4.2	Legendre's Equation	195
4.3	Frobenius' Method	201
4.4	Bessel's Equation I	207
4.5	The Second Solution	214
4.6	Bessel's Equation II	222
4.7	The Point at Infinity; Asymptotic Series	227
4.8	A Convergence Proof	234
	Notes and References	238
	Miscellaneous Exercises	238

## 5 Laplace Transform

---

5.1	Introduction	242
5.2	Applications to Initial Value Problems	248
5.3	Further Applications of the Laplace Transform	257
5.4	Discontinuity, Shift, and Impulse	264
5.5	The Convolution	275
5.6	Applications to Control Theory	282
5.7	Periodic Functions	292
	Notes and References	302
	Miscellaneous Exercises	303
	Appendix: Partial Fractions	305

## 6 Numerical Methods

---

6.1	Elementary Methods	314
6.2	Analysis of Error	323
6.3	Runge-Kutta Methods	330
6.4	Predictor-Corrector Methods	340
6.5	Stability and Step Length	346
	Notes and References	352
	Computer Programs	352
	Miscellaneous Exercises	355

## 7 Systems of Linear Differential Equations

---

7.1	Introduction	357	
7.2	Elimination Method	362	
7.3	Eigenvalues and Eigenvectors	368	
7.4	Homogeneous Systems with Constant Coefficients		377
7.5	Theory of Linear Systems	385	
7.6	Nonhomogeneous Systems	394	
7.7	Qualitative Behavior	404	
	Notes and References	409	
	Miscellaneous Exercises	410	

## 8 Nonlinear Second-Order Equations

---

8.1	Introduction	414	
8.2	Critical Points of Linear Systems	423	
8.3	Stability by Linear Comparison	435	
8.4	Stability by the Direct Method	443	
8.5	Limit Cycles	454	
8.6	Exact Solution of Nonlinear Equations		464
	Notes and References	473	
	Miscellaneous Exercises	474	

## 9 Boundary Value Problems

---

9.1	Two-Point Boundary Value Problems	477	
9.2	Eigenvalue Problems	487	
9.3	Singular Problems	496	
9.4	Green's Functions	501	
9.5	Eigenfunction Series: Two Examples	511	
9.6	Eigenfunction Series: Sturm-Liouville Problems		521
9.7	Other Eigenfunctions Series	529	
9.8	Numerical Methods	541	
	Notes and References	551	
	Miscellaneous Exercises	552	

## 10 Partial Differential Equations

---

10.1	Introduction	555	
10.2	The Homogeneous Heat Problem	564	
10.3	Examples	571	
10.4	The Homogeneous Wave Problem	578	
10.5	Nonhomogeneous Problems	588	

10.6	The Potential Equation	597
10.7	Other Potential Problems	607
10.8	Other Coordinate Systems	615
10.9	Characteristics and Classification of Equations	622
	Notes and References	629
	Miscellaneous Exercises	630

## Appendix A Matrix Algebra

---

A.1	Basic Algebra of Matrices	A1
A.2	Matrix Multiplication	A9
A.3	Elimination	A15
A.4	Inverse	A23
A.5	General Systems	A28
A.6	Rank	A35
A.7	Determinant	A41

## Appendix B Mathematical References

---

Trigonometry	A48
Complex Numbers	A48
Polynomials	A50
Determinants	A51
Calculus	A54
Series	A55

Bibliography	A57
Answers to Odd-Numbered Exercises	A59
Index	A121